CHAPTER TWO

GROUNDWATER MOVEMENTS

2.1 INTRODUCTION

2.1.1 FORCES, PRESSURE, ENERGY AND GROUND WATER

Force (F) is the product of **mass (m) and acceleration (a)**:

$$F = ma = kg \cdot m / s^2 = N = newton. \tag{5.1}$$

Mass is the amount of matter in a volume. The mass of concern is water, and the acceleration of concern is due to **gravity (9.81 m/s²)**. The force of **gravity** pulls water down and keeps it flowing. Ground water does not flow freely through pore space because of frictional resistance. Friction occurs where the surfaces of water molecules flow across the surfaces of sediment or rock.

Pressure (P) is the quotient of **force divided by area:**

$$P = \frac{Force}{area} = N / m^2 = Pa = pascal. \tag{5.2}$$

Pressure acts on the zone of saturation and is provided by the collective stress of geologic material (sediment, rock, water) and atmospheric pressure.

A thin film of water tends to "stick" to an object after drainage is complete. This is due to **adhesion** and **cohesion (surface tension)** when water is exposed to air. Adhesion is the molecular attraction between water and the surface of another object; cohesion is the attraction between water molecules.

Viscosity is the internal resistance of a liquid to a shearing force or flow. In regard to ground water, viscosity is primarily the result of interaction between water molecules. It can be thought of as the "internal friction within the liquid". The viscosity of water increases as water temperature decreases.

Energy is the capacity to do work. **Work** is the displacement of an object through a certain distance by a force. Every drop of water has the ability to do work, and therefore, contains energy. **Gravitational potential energy (PE)** is the energy possessed by an object by virtue of its position above a datum plane, and it can be expressed as

$$PE = mgh = J = joule \tag{5.3}$$

and defined as

The work required to lift a mass from a datum to its current position or the work it can do when falling from its current position to the datum.

For consistent units,

m = mass of an object = M
g = acceleration due to gravity = L / T^2
h = height of the object above a datum plane = L.

Kinetic energy (KE) is the energy in an object in motion:

$$KE = \frac{1}{2} mv^2 = J = \text{joule} \quad (5.4)$$

where

v = velocity = L / T.

Ground water possesses potential energy relative to its position above a datum, and converts this energy into thermal and kinetic energy as it flows down slope. Gravity is the force that drives ground water through aquifers. Municipal water towers are analogous to natural confined aquifers. As demonstrated in Figure 5.1, ground water flows from areas of high potential energy to areas of low potential energy --- and when discharging --- does so from areas of high pressure into areas of low pressure.

2.1.2 HYDRAULIC HEAD

The height of a column of water above a datum plane is called **hydraulic head** or simply **head**. When considering the applied study of ground water, **head is the elevation of water in a well**, and mean sea level is generally used as datum. This is also known as the **static water table**. Head as referred to here is the **total head (H_T)**. Ground water flows in the direction of decreasing total head, which has three components:

1) Elevation head, h_e = L, which is measured from sea level to a point at the bottom of the well.
2) Pressure head, h_p = L, which is measured from the point at the bottom of the well to the top of the water level in the well.
3) Velocity head, h_v = L, which, in regard to the study of ground-water flow, is often negligible, and therefore neglected.

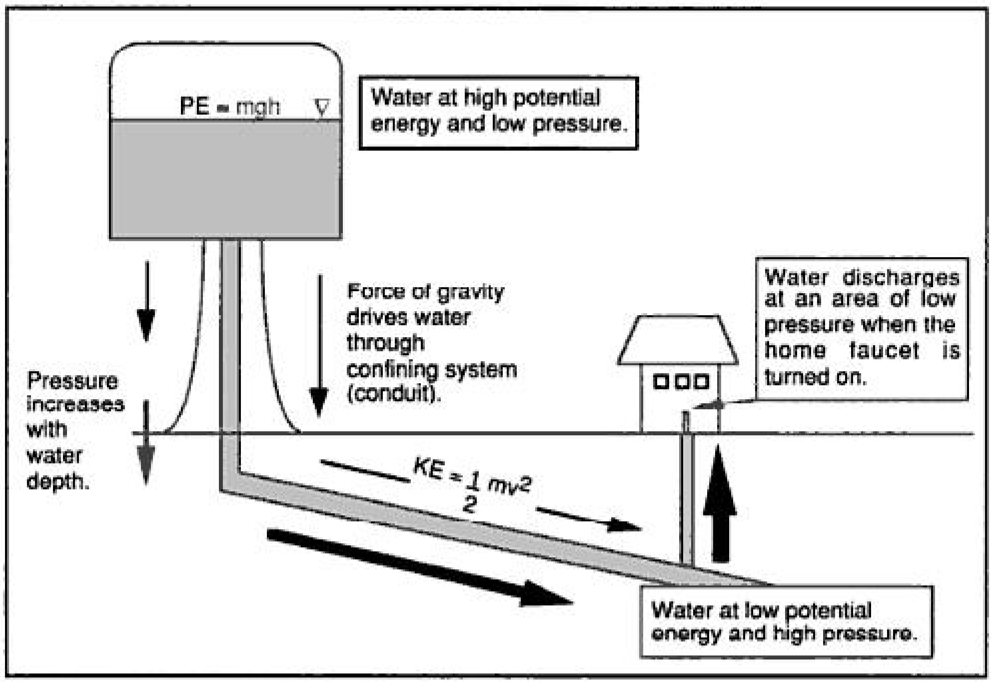

Figure 5.1. City water system as an artificial confined aquifer. An analogous demonstration of the principle of ground-water flow.

Total head can be expressed as

$$H_T = h_e + h_p + h_v. \tag{5.5}$$

Hydraulic head of water is the energy of water per unit weight (ρg).

Elevation head (h_e) is a form of potential energy, which again, is energy possessed by an object relative to its position above a datum plane. Potential energy can be expressed as

$$PE = mgh \tag{5.6}$$

and as a form of potential energy elevation head can be expressed as

$$PE = \rho g h_e, \tag{5.7}$$

where density (ρ) replaces mass (m) in equation (5.6). The unit weight of a volume of water is

$$\gamma = \rho g; \tag{5.8}$$

therefore, by the definition of hydraulic head, **energy of water per unit weight**, the elevation head can be defined as (Fig 5.2)

$$\frac{PE}{\rho g} = \frac{\rho g h_e}{\rho g} = \frac{\gamma h_e}{\gamma} = h_e. \tag{5.9}$$

Pressure head, (h_p), is also a form of potential energy. For a liquid at rest, the pressure (P) at any depth below a water surface can be defined as

$$P = \rho g h_p. \tag{5.10}$$

where h_p = depth below a water surface or the height of a column of water above the measured point. Rearranging equation (5.10) results in pressure head (Fig. 5.3):

$$\frac{P}{\rho g} = \frac{\rho g h_p}{\rho g} = \frac{\gamma h_p}{\gamma} = h_p. \tag{5.11}$$

Water that is moving can do work. The erosion of treeless hills and transport of sediment are forms of work by water; therefore, water that is moving possesses kinetic energy (KE). Again,

$$KE = \frac{1}{2} m v^2. \tag{5.12}$$

Substituting the density of water for mass in equation (5.12), results in the amount of kinetic energy in water that is flowing:

$$\frac{1}{2}\rho v^2 = \rho g h_v. \tag{5.13}$$

Again, by the definition of hydraulic head: **energy of water per unit weight:**

$$h_v = \frac{\frac{1}{2}\rho v^2}{\gamma} = \frac{\frac{1}{2}\rho v^2}{\rho g} \tag{5.14}$$

or

$$h_v = \frac{v^2}{2g}. \tag{5.15}$$

Therefore, the total hydraulic head can be expressed as

$$H_T = \frac{\rho g h_e}{\rho g} + \frac{\rho g h_p}{\rho g} + \frac{v^2}{2g}. \tag{5.16}$$

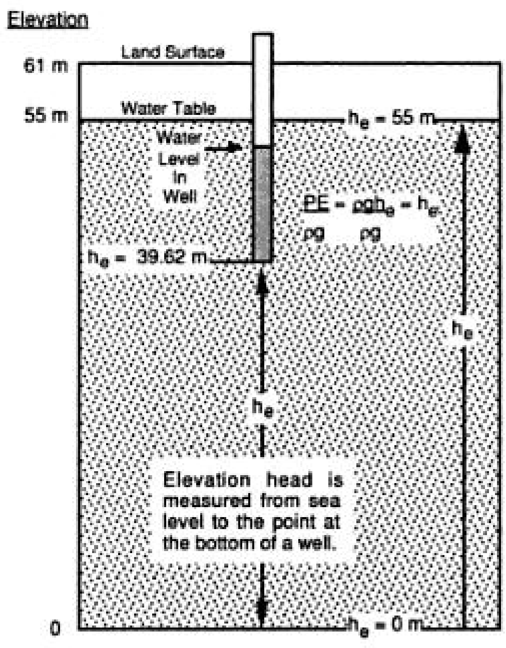

Figure 5.2. Elevation head measured in a well and at the water table.

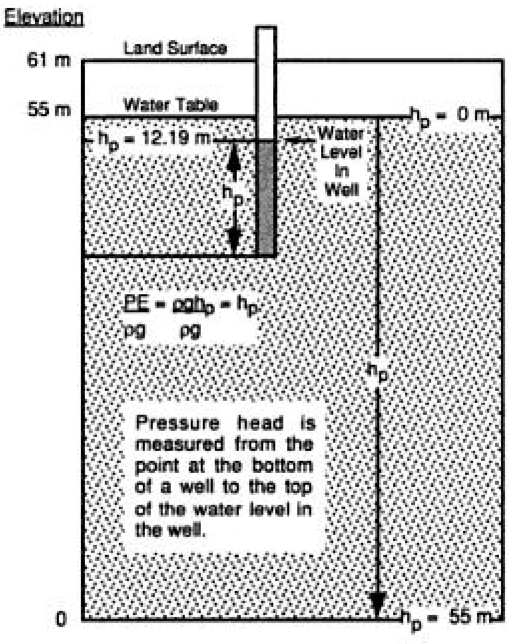

Figure 5.3. Pressure head measured in a well and at the water table.

Most ground water flows through sediment in laminar fashion, that is, streamlines are smooth, straight and parallel (Fig. 5.4). Here, ground-water flow is generally non-turbulent. The rate of flow is slow enough to make velocity head negligible; therefore (Fig. 5.5),

$$H_T = \frac{\rho g h_e}{\rho g} + \frac{\rho g h_p}{\rho g} \tag{5.17}$$

or simply

$$H_T = h_e + h_p. \tag{5.18}$$

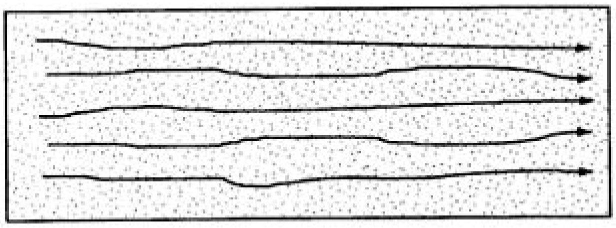

Figure 5.4. Laminar flow through a ground-water system.

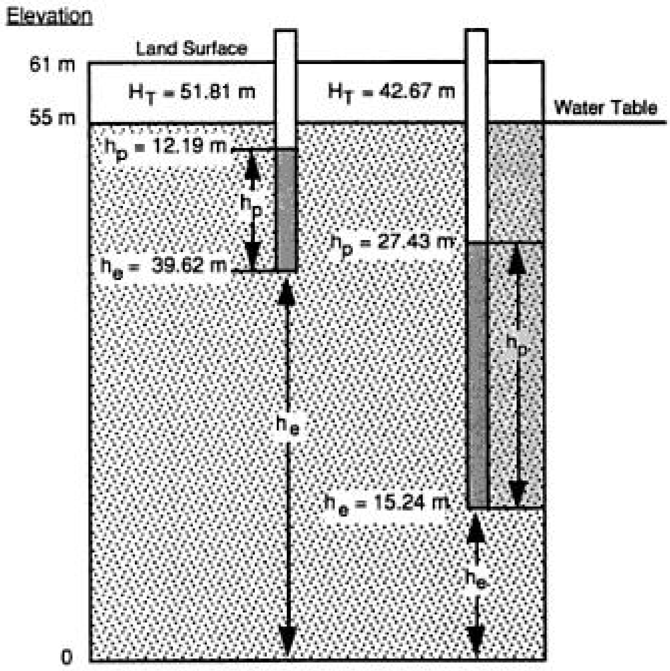

Figure 5.5. Elevation, pressure and total head measured in two wells.

Multiplying the total head (H_T) by the acceleration of gravity (g) results in **force potential (Φ)** (Hubbert, 1940) (Fig. 5.6):

$$\Phi = gH_T = g(h_e + h_p + h_v) = gh_e + gh_p + gh_v = L^2 / T^2. \tag{5.19}$$

Neglecting velocity head,

$$\Phi = gH_T = g(h_e + h_p) = gh_e + gh_p. \tag{5.20}$$

The hydraulic head of water is the potential energy of water **per unit weight**. Force potential is the potential energy of water **per unit mass**, and is the driving force behind ground-water flow. It is the potential energy of ground water measured at a point in the aquifer.

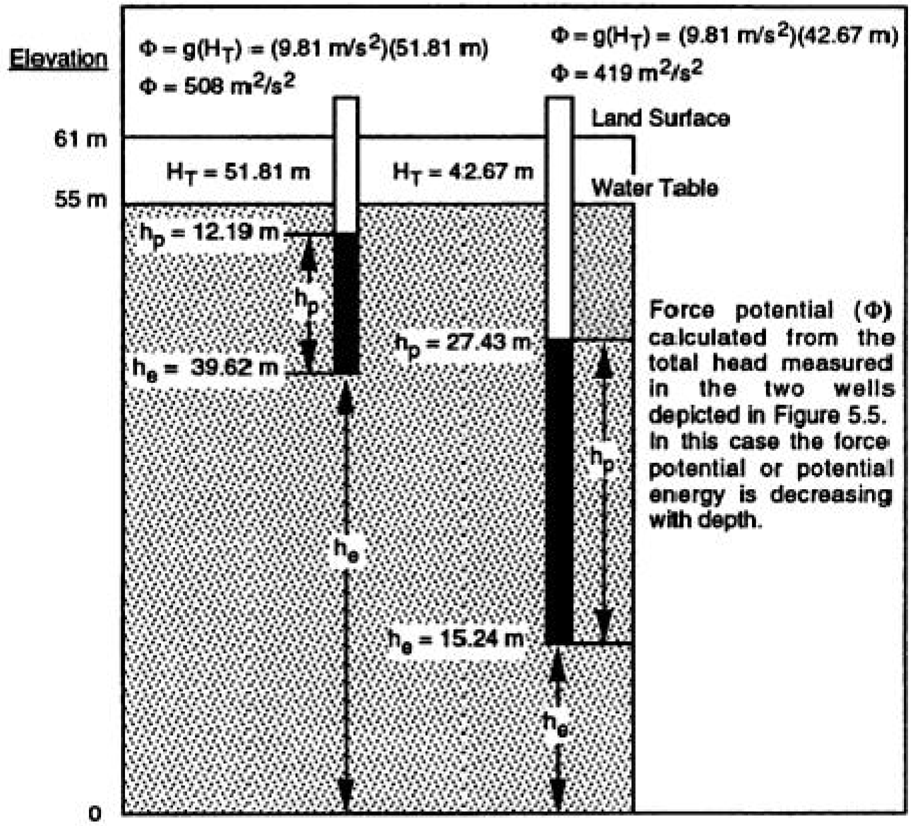

Figure 5.6. Force potential (Φ) calculated from total head. Potential energy is decreasing with depth.

2.1.3 CHANGE IN HEAD

Consider that the two wells in Figures 5.5 and 5.6 have been constructed about 2 feet apart, and that the heads were measured in about a minute of time. One head is obviously lower than the other. In this case, potential energy in regard to the two wells decreases with ground-water depth. If the wells are only 2 feet apart, why has the measured head significantly decreased between the two wells? Remember, ground water is flowing. Most of the energy loss (Δh) is due to friction. Potential energy is converted to heat energy as ground water flows through narrow tortuous paths between sediment. It begins to flow from a point of maximum head or potential energy. As ground water flows some of the potential energy is also converted to kinetic energy. This conversion from one form of energy to other forms is the measured head loss or change in head (Δh) between the two wells. An apparatus constructed by Allen Hazen (1892), in his attempts to determine permeability of some soil, can be used to demonstrate the above argument (Fig. 5.7). Ground-water flow through the apparatus is vertical and so is the flow in Figures 5.5 and 5.6. The Δh is due to the conversion of potential energy, to thermal and kinetic energy, as water flows downward to the outlet where it discharges at an area of low pressure.

2.1.4 POTENTIALS AND GROUND-WATER FLOW

To review, most ground-water models divide the subsurface into three zones. The **upper zone** is referred to as the **unsaturated or vadose zone**. Ground water in the upper zone is generally dominated by forces of adhesion and cohesion. The **capillary fringe** is an area contained in both the upper and lower zones, however, water in the fringe is also under the influence of surface tension. The **lower zone** is usually identified as the **saturated or phreatic zone**. Ground water in this zone, at and below the water table, is gravity driven. Most aquifers are located in the phreatic zone (the exception are perched aquifers). The previous discussion considered gravity driven ground-water flow in an isotropic, homogeneous zone of saturation --- and it will continue in that vein.

Potential energy decreases with depth in Figure 5.6; therefore, ground water must have moved or flowed from high potential energy to low potential energy. This is the case of a ground-water recharge area. **Ground-water recharge** is an area where water enters an aquifer system. Another case can be constructed where ground water moves from low potential energy to high potential energy, that is, an area where potential energy increases with depth (Fig. 5.8). This is the case of a ground-water discharge area. **Ground-water discharge** is where ground water leaves or flows out from an aquifer. We can now establish some rules for energy potential and ground-water flow (Figs. 5.9 and 5.10):

1) Ground water flows from high potential energy to low potential energy or in the direction of decreasing total head (H_T).
2) Recharge areas are where potential energy decreases with depth.
3) Discharge areas are where potential energy increases with depth.

4) An equipotential line is a line connecting points of equal hydraulic head.
5) Ground-water streamlines are constructed perpendicular to equipotential lines.
6) Ground-water streamlines diverge at areas of recharge.
7) Ground-water streamlines converge at areas of discharge.
8) Equipotentials tend to be perpendicular to the face of a geologic boundary, and ground-water flow tends to parallel that boundary.
9) Equipotentials tend to be parallel to the face of a constant head boundary and ground-water flow tends to be perpendicular to that boundary.
10) Streamlines are at an oblique angle to the water table when recharge or discharge occurs.
11) Streamlines refract across geologic beds with different K values.
12) A ground-water divide is a no flow boundary.

The term **equipotential** is depicted in Figure 5.9. An **equipotential line** connects points of equal hydraulic head, that is, the total hydraulic head is the same for all points along this line. Equipotential lines are constructed by interpolating known hydraulic heads measured in wells surveyed in relation to mean sea level. Wells constructed on the same equipotential will have water levels of equal height above datum.

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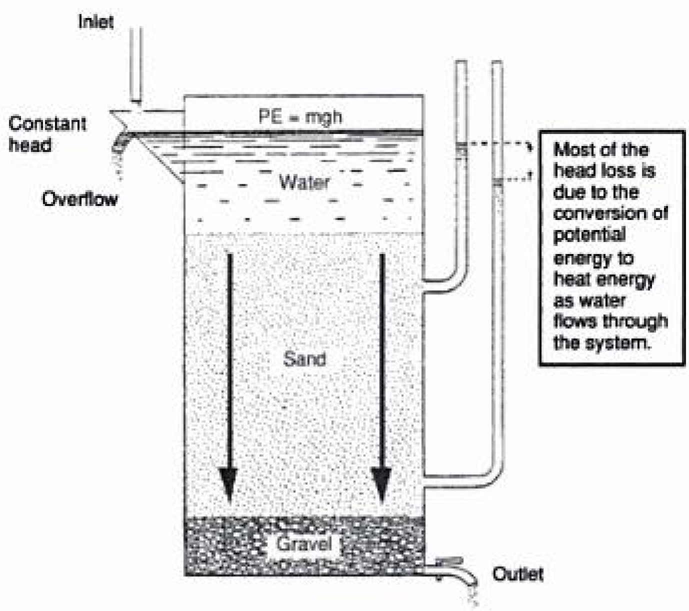

Figure 5.7. Hazen's experimental filter for the determination of the laws of flow of water through sand (modified from Slichter, 1902).

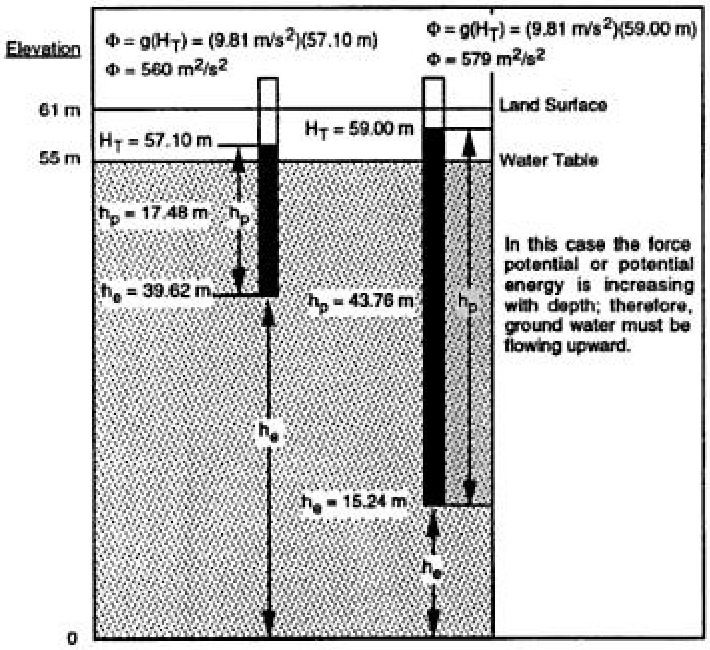

Figure 5.8. Potential energy increasing with depth.

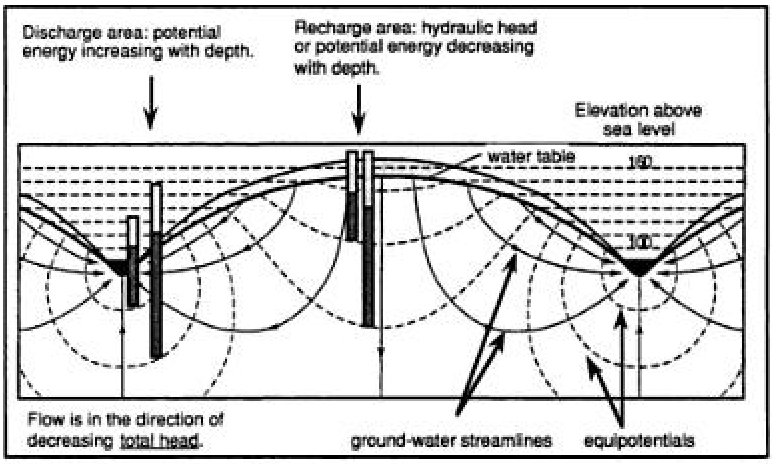

Figure 5.9. Ground-water flow in an isotropic, homogeneous water table aquifer (cross-sectional view) (modified from Hubbert (1940), *Journal of Geology*, v. 48, n. 8 reprinted with permission of the University of Chicago Press).

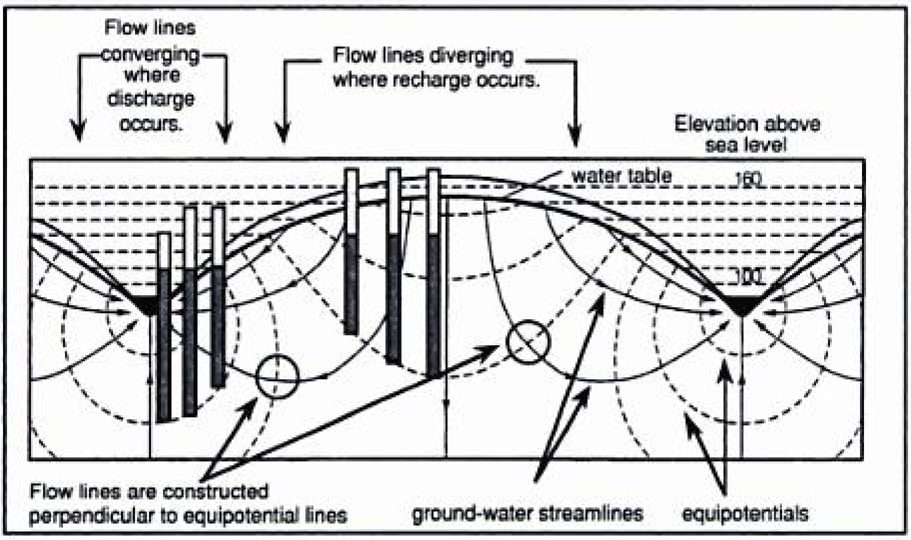

Figure 5.10. Cross-section of flow net showing relationship between equipotentials and ground-water streamlines.

2.1.5 EQUIPOTENTIALS AND BOUNDARY CONDITIONS

There are essentially three principal types of aquifer boundaries: a no flow boundary, a constant head boundary, and a free surface boundary, when considering unconfined aquifers. A **no flow boundary** is just that, a ground-water boundary across which no ground-water flow occurs. The two common types are an **impermeable bedrock boundary** and a **ground-water divide boundary**. For a homogeneous, isotropic system, equipotentials are perpendicular to the face of the boundary, and ground-water flow is parallel to the boundary (Fig. 5.11 and 5.12). A **constant head boundary** is a surface of equal head having the same head value at all points.

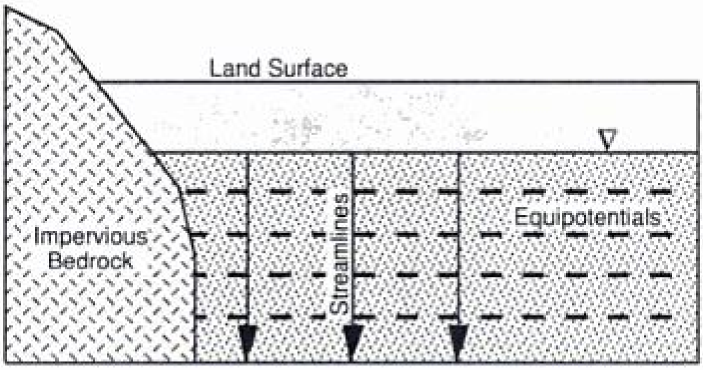

Figure 5.11. A no flow boundary.

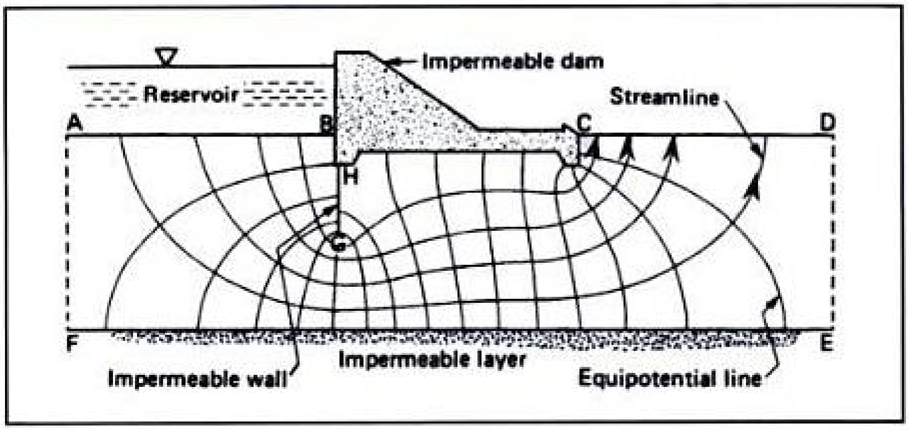

Figure 5.12. Lines AB and CD are constant head boundaries. Lines EF and BGHC are no flow boundaries (Franke, Reilly and Bennett, 1987).

This occurs where an aquifer outcrops beneath a lake or river. The surface-water stage is nearly uniform over all points of the outcrop, and does not vary appreciably with time (Franke, Reilly and Bennett, 1987). For a homogeneous and isotropic system, equipotentials tend to be parallel to the face of the boundary, and ground-water flow tends to be perpendicular to the boundary (Figs. 5.12 and 5.13). A common **free surface boundary** is the **water table**, which is the upper surface of an unconfined aquifer. The position of the water table is not fixed and varies with time. Streamlines are at an oblique angle to the water table when recharge or discharge occurs across this boundary. If there is no flow across the water table, then streamlines are generally parallel to it (Figs. 5.14 and 5.15).

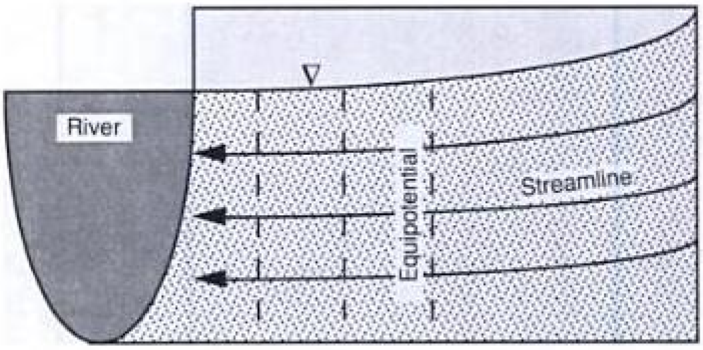

Figure 5.13. A constant head boundary.

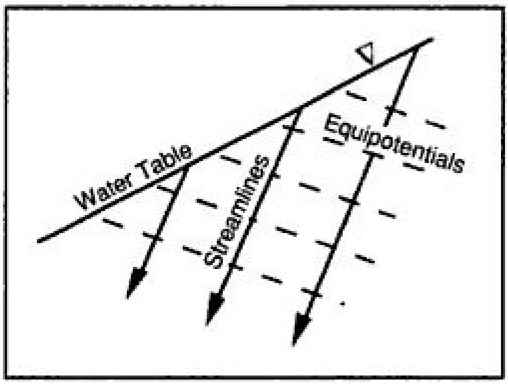

Figure 5.14. Free surface or water table boundary with recharge occurring.

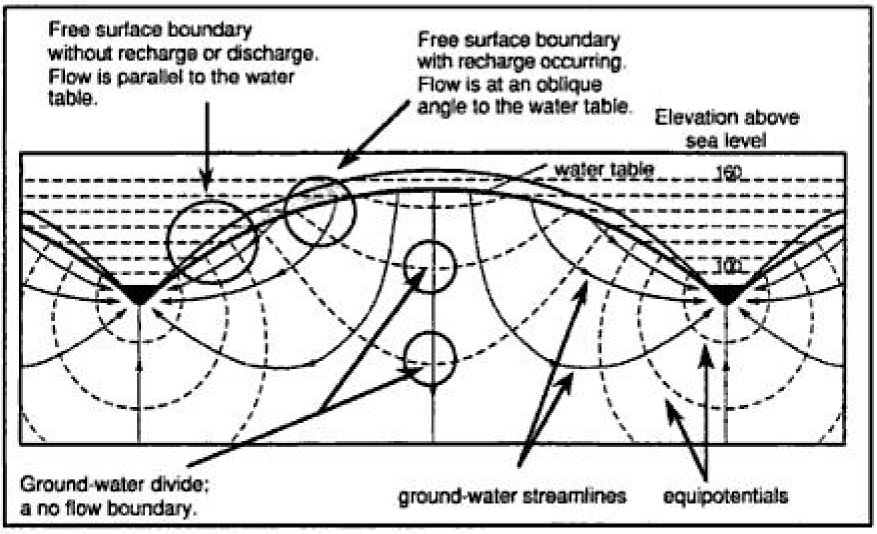

Figure 5.15. Example of rules concerning equipotentials and streamlines.

2.1.6 EQUIPOTENTIALS AND STRATIFICATION

Most aquifer systems consist of aquifers and confining beds; therefore, in these systems, ground water flows through aquifers and across confining beds. Streamlines refract or bend at these boundaries, because hydraulic conductivities (K) of aquifers are much greater when compared to confining units, that is, frictional resistance to flow is much less in aquifers. Because resistance to flow is less in aquifers, equipotentials tend to be perpendicular to a confining bed; whereas, streamlines tend to be parallel to the confining bed. As ground water flows from an aquifer into a confining bed, refraction or bending of streamlines occurs in a direction perpendicular to the boundary. As streamlines move from the confining bed into another aquifer unit, they are refracted back toward a direction that parallels the boundary (Fig. 5.16 and 5.17). The angles of refraction (θ) are proportional to the differences in hydraulic conductivities (K), and can be expressed by equation (5.21):

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{K_1}{K_2}. \tag{5.21}$$

When considering confined aquifers, equipotentials tend to be perpendicular to the boundaries, while flow tends to be parallel (Fig. 5.17).

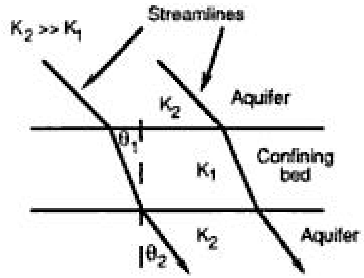

Figure 5.16. Refraction across beds with different K values.

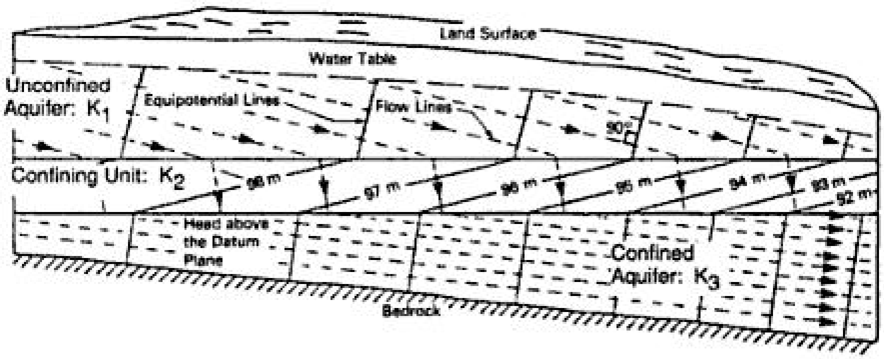

Figure 5.17. Example of refraction across beds with different K values. Numbers are head values (modified from Heath, 1983).

2.1.7 FLOWING WELLS

Ground water flowing in aquifers is under pressure greater than atmospheric. Ground-water flow occurs from areas of high potential energy to areas of low potential energy, or in the direction of decreasing total head. When ground water discharges, it discharges from an area of high pressure to an area of low pressure. A well penetrating an aquifer creates an area of low pressure into which ground water under pressure flows. When pressure is great enough, upward flow may reach the surface: this is a flowing well. Essentially two conditions can produce flowing wells:

1) A well tapping water under pressure in a confined aquifer (Fig. 5.18).
2) A well tapping an unconfined aquifer near an area of ground-water discharge, because this is where potential energy increases with depth (Fig. 5.19).

In the artesian case, water flows upward, but many wells tapping such aquifers may not produce flowing wells, because the water does not reach the surface. They are still artesian wells, but they are not flowing wells. The height that water will rise in an artesian well is equal to the height of the area of recharge minus energy converted to thermal and kinetic energy. Thermal energy due to friction occurs between water molecules, and sediment and casing, as water moves through the aquifer and up the well.

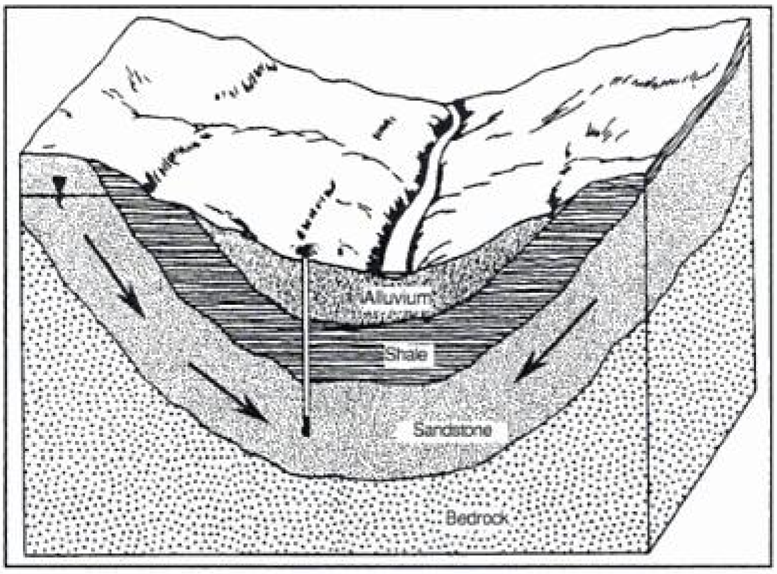

Figure 5.18. Flowing well in a confined aquifer (modified from Riewe (1992) *Water Well Journal*, June, reprinted with permission of the Ground Water Publishing Company).

In the case of a water table or unconfined aquifer, the deeper a well point penetrates an area of discharge, the greater the potential energy; therefore, the deeper the well the higher the head. When the head reaches the surface the well is flowing (Fig. 5.19). In one project experienced by the author, the head in a well tapping a discharge area reached a mean elevation of 4.5 feet above ground surface. The well was constructed at a depth of only 20 feet.

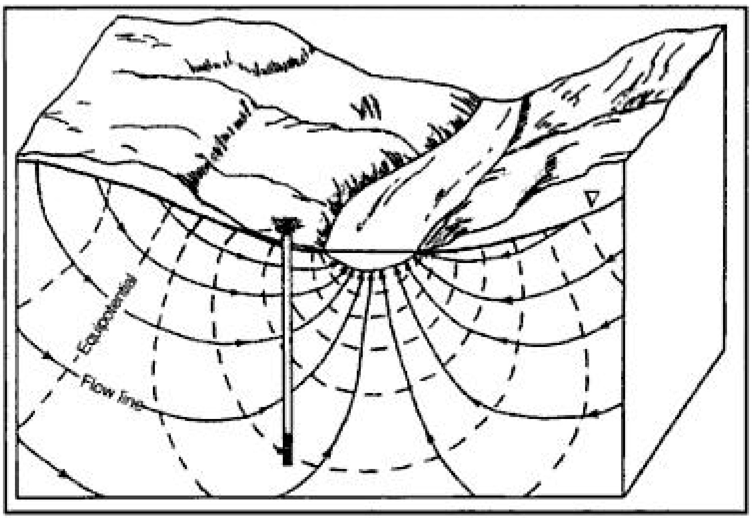

Figure 5.19. Flowing well in an unconfined aquifer (modified from Riewe (1992) *Water Well Journal*, June, reprinted with permission of the Ground Water Publishing Company).

2.1.8 THE HYDRAULIC GRADIENT

A reduction or change in head (Δh) occurs between two points (L) in the direction of ground-water flow. As indicated in Section 5.3 and Figure 5.7, most of the head change results from the conversion of potential energy to heat energy as ground water moves through sediment and up the well casing. Figure 5.7 demonstrated loss of head in regard to vertical flow. **Horizontal hydraulic gradient** occurs in both confined and unconfined aquifers. This can be demonstrated using the apparatus depicted in Figure 5.20, through which a **constant, steady, horizontal flow** is occurring. As water moves through the sand under a hydraulic head at one end, the pressure dissipates from the maximum value at the end at which water enters, to a minimal value at the free end from which water flows (Slichter, 1902). Flow is horizontal; therefore, no change in elevation head is occurring as water flows from "A" to "B". Head is constant and flow is steady, which means water leaves the system at the same rate it enters the system; therefore, kinetic energy must also be constant. Yet a change in pressure energy can be measured at each piezometer (a small diameter well), and is represented by decreasing head. Again, this change in head is due mostly to the conversion of potential energy to thermal energy. Because no change in elevation head is occurring, the conversion of potential energy to thermal energy results in a decrease in pressure head. Even though water is free to escape at point "B", the pressure gradient is sufficient to produce flow between points "AB". The pressure gradient is the **hydraulic gradient (Δh/L)**. The hydraulic gradient is the slope of the water table or potentiometric surface (Fig. 5.21). Sloping water tables are flowing water tables. Water table contours tend to mimic topographic contours, but the slope tends to increase with increasing precipitation and decrease during dry periods (Fig. 5.22).

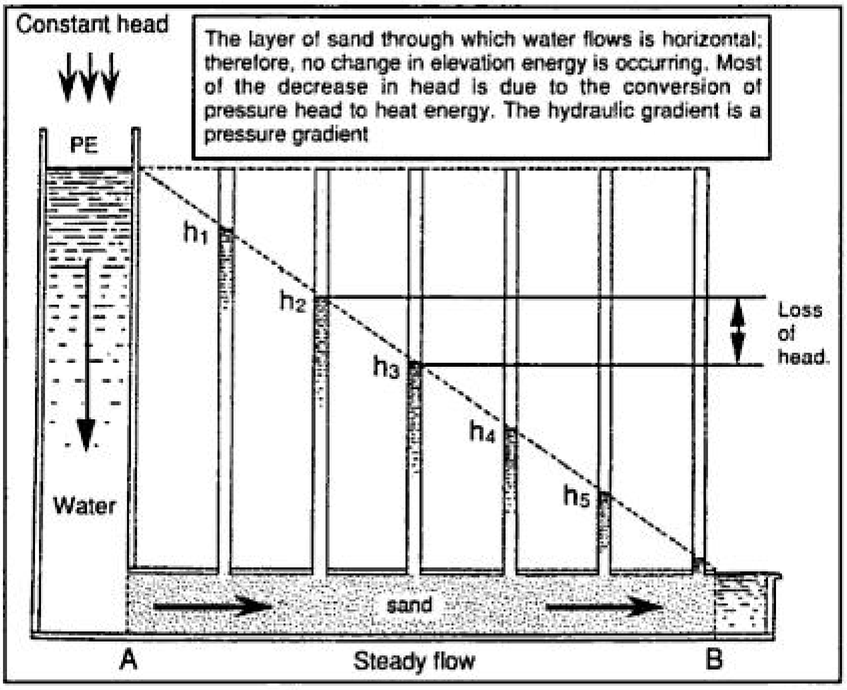

Figure 5.20. Apparatus demonstrating the decline in pressure as water flows through pervious material (modified from Slichter, 1902).

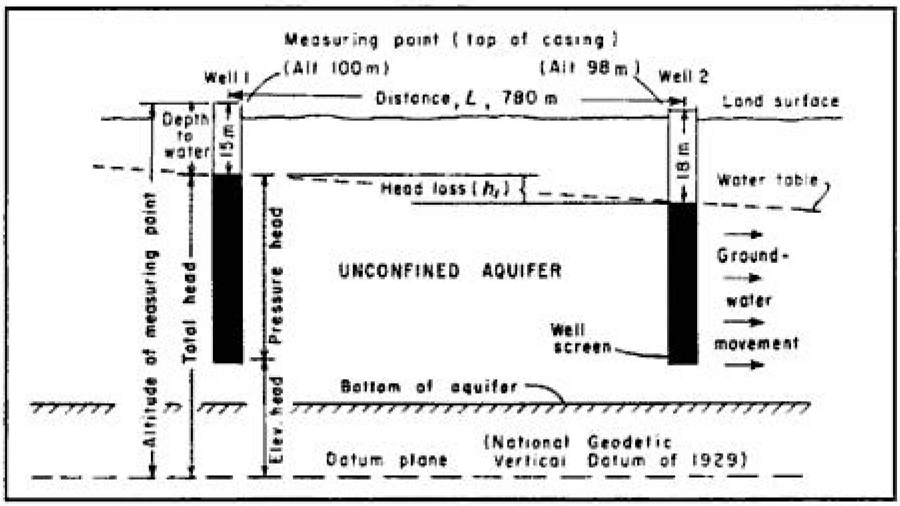

Figure 5.21. Hydraulic gradient in an unconfined aquifer (Heath, 1983).

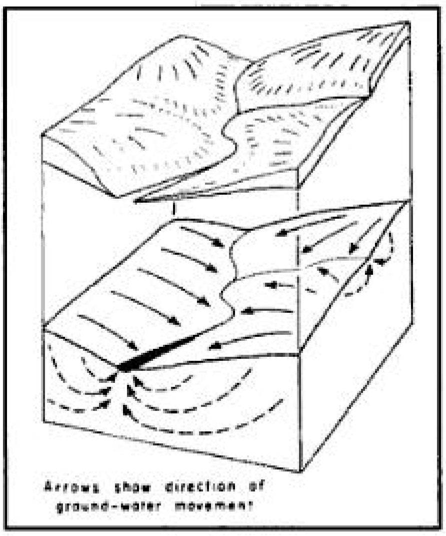

Figure 5.22. Ground-water flow in relation to topography (Heath, 1983).

The hydraulic gradient and direction of ground-water flow can be determined if the following data are available for three wells located in a triangular arrangement (Fig. 5.23) (Heath, 1983):

1. The surveyed and geographic position of wells.
2. The distance between the wells.
3. The total head (H_T) at each well.

2.1.9 Piezometric Head h [L]

There is a well-defined relationship between the piezometric head at a point in an aquifer and the hydrostatic pressure at that point. For a fluid of uniform density, the hydrostatic pressure, *P* [$ML^{-1} T^{-2}$] at a point is given by:

$$P = \rho g H \qquad \textbf{(2.1)}$$

where:

H is the height of the column of fluid above the point in question [L]
ρ is the density of the fluid [ML^{-3}]
g is the acceleration due to gravity [LT^{-2}]

The height of the column of fluid **H** can be used as a measure of the hydrostatic pressure, but piezometric head **h** is measured above an arbitrary datum and is given by:

$$h = \frac{P}{\rho g} + z \qquad \textbf{(2.2)}$$

where: **z** is the elevation of the point above the datum or reference level [L]

Sometimes ***P/ρg*** is referred to as the pressure head, and ***z*** as the elevation head (see **Figure 2.1**).

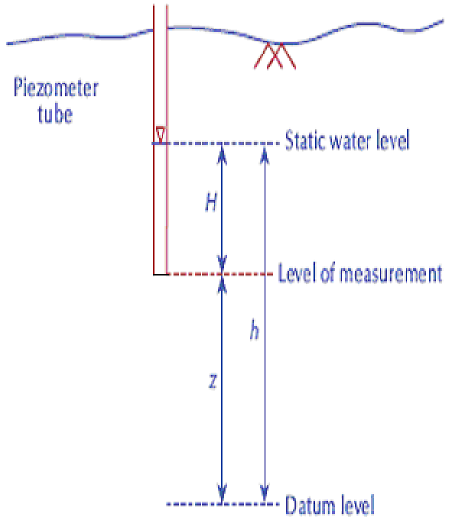

Figure 2.1 Piezometric head

2.1.10 Darcy's Law for Flow in Porous Media

General

The classic work on the flow of water through a porous medium was conducted by Henri Darcy in France in 1856. Darcy's result is of fundamental importance and remains at the heart of almost all groundwater flow calculations.

Darcy discovered that the discharge **Q** of water through a column of sand is proportional to the cross sectional area **A** of the sand column, and to the difference in piezometric head between the ends of the column, **h1– h2**, and inversely proportional to the length of the column **L**. That is:

$$Q = KA\frac{h_1 - h_2}{L} \qquad \textbf{(2.3)}$$

Darcy's experiment is shown schematically in **Figure 2.2**. The constant of proportionality *K* is known as the hydraulic conductivity [LT^{-1}]. The implication here is that the specific discharge is proportional to the applied force. Darcy's experiments were one-dimensional. In this section, we generalize the results of the experiments to give Darcy's Law in three dimensions.

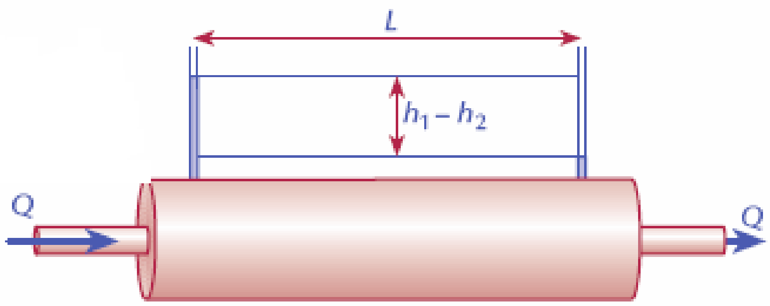

Figure 2.2 A schematic diagram of Darcy's experiment

Rather than referring to the total discharge **Q**, it is often more convenient to standardize the discharge by considering the volume flux of water through the column, i.e. the discharge across a unit area of the porous medium. In the context of groundwater, the volume flux is called the **specific discharge q** [LT $^{-1}$] and is given simply by **Q/A**. Darcy's result can then be written in terms of the specific discharge and the difference in head between the ends of the column.

$$q = \frac{Q}{A} = -K\frac{h_2 - h_1}{L} \qquad \textbf{(2.4)}$$

The fraction $\frac{h_2 - h_1}{L}$ is called the average hydraulic gradient over the length of the column. As **L** tends to zero, the average hydraulic gradient becomes an increasingly close approximation to the point value of the derivative of head with respect to distance *x*.

Darcy's experimental result then becomes:

$$q = -K\frac{dh}{dx} \qquad \textbf{(2.5)}$$

which describes Darcy's Law at any point in the porous medium. The spatial derivative of head dh/dx is called the hydraulic gradient at that point. There are two important points to note:

- If the hydraulic gradient is positive, the specific discharge is negative. This reflects the fact that the groundwater moves from high to low head. So, for example, since the water table in **Figure 2.3** slopes upwards away from the origin (i.e. $dh/dx > 0$), the water moves back towards it (i.e. $q < 0$).

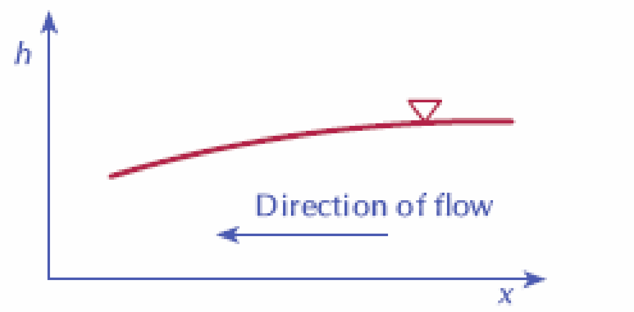

Figure 2.3

- Although we have referred to Darcy's Law at a point, specific discharge, hydraulic conductivity and hydraulic gradient can be defined only as averages taken over a volume of rock. The assumption implicits in everything that follows is that this volume is small in comparison with the scale of any problem under consideration. The volume will vary in size depending upon the scale of the problem. For example, at the scale of a study in the laboratory the value of hydraulic conductivity at a point will be taken as an average over a few cubic centimetres, whereas at the regional scale the point hydraulic conductivity may be an average taken over hundreds of cubic metres which may include a variety of different rock formations.

Specific Discharge and Groundwater Velocity

There is a fundamental relationship between specific discharge and groundwater velocity. Specific discharge has the dimensions of velocity, and in some books it is referred to as the Darcy velocity.

This terminology is misleading and is best avoided, as the specific discharge is **not** a velocity – and it is certainly not the same as the groundwater velocity. To illustrate the difference, consider what happens when we pump water through an empty pipe.

The relationship between discharge, cross-sectional area and water velocity **v** is **v = Q/A,** and in this case the velocity *is* equal to the specific discharge.

However, if we repeat the experiment but this time fill the pipe with sand, the cross-sectional area of the pipe remains the same, but the cross sectional area that is open to flow is much reduced, and so, for the same discharge (and hence the same specific discharge) through the pipe, the water will be forced through a smaller cross-sectional area and will, therefore, have to travel faster than if the pipe was empty. This means that the water velocity will be higher than the specific discharge.

It can be shown that the effective area open to flow is **An_e**, where **n_e** is the effective porosity of the rock, and hence the groundwater velocity can be calculated by:

$$u = \frac{q}{n_e} \quad \textbf{(2.6)}$$

Note that this does not mean that water travels more easily through low porosity rock. It does mean that if the specific discharge through two rocks is the same, then the water will travel faster through the rock with the lower effective porosity**.**

Validity of Darcy Law

In applying Darcy's law it is important to know the range of validity within which it is applicable. Because velocity in laminar flow is proportional to the first power of the hydraulic gradient, it seems reasonable to believe that Darcy's Law applies to laminar flow in porous media.

Experiments show that Darcy's law is valid for $N_R < 1$ and does not depart seriously up to $N_R = 10$. This, then, represents an upper limit to the validity of Darcy's law. Where, N_R is the Reynolds number that is expressed as

$$N_R = \frac{\rho v D}{\mu} \quad \textbf{(2.7)}$$

where ρ is the fluid density, v the velocity, D the diameter and μ the viscosity of the fluid.

Fortunately, most natural groundwater flow occurs with $N_R < 1$ so Darcy's law is applicable. Deviations from Darcy's law can occur where steep hydraulic gradients exist, such as near pumping wells; also, turbulent flow can be found in rocks such as basalt and limestone that contain large underground openings. It should also be noted that investigations have shown that Darcy's law may not be valid for very slow water flow through dense clay.

2.2 Heterogeneity and Anisotropy of Aquifers

2.2.1 Introduction

Hydraulic conductivity values usually show variations through space within a geologic formation. They may also show variations with the direction of measurement at any given point in a geologic formation. The first property is termed **heterogeneity** and the second **anisotropy**. The evidence that these properties are commonplace is to be found in the spread of measurements that arises in most field sampling programs. The geological reasoning that accounts for their prevalence lies in an understanding of the geologic processes that produce the various geological environments. To summarize,

- An aquifer is **homogeneous** if its hydraulic properties are the same at any point in space. Non-homogeneous aquifers are said to be **heterogeneous**.

➢ An aquifer is **isotropic** if its hydraulic properties are the same in any direction in space. Aquifers that are not isotropic are said to be **anisotropic**.

2.2.2 Homogeneity and Heterogeneity

If the hydraulic conductivity **K** is *independent* of position within a geologic formation, the formation is **homogeneous**. If the hydraulic conductivity **K** is *dependent* on position within a geologic formation, the formation is **heterogeneous**. If we set up an xyz coordinate system in homogeneous formation, **K**(x,y,z) =**C**, **C** being a constant; whereas in a heterogeneous formation, **K**(x,y,z)≠**C.**

2.2.3 Isotropy and Anisotropy

If the hydraulic conductivity **K** is *independent* of the direction of measurement at **a point** in a geologic formation, the formation is **isotropic** at that point. If the hydraulic conductivity **K** *varies* with the direction of measurement at **a point** in a geologic formation, the formation is **anisotropic** at that point. if an xyz coordinate system is set up in such a way that the coordinate directions coincide with the principle direction of anisotropy, the hydraulic conductivity values in the principle directions can be specified as K_x, K_y, and K_z. At any point (x,y,z), an isotropic formation will have $\mathbf{K_x= K_y= K_z}$, whereas an anisotropic formation will have $\mathbf{K_x \neq K_y \neq K_z}$ (see **Figure 2.4**)

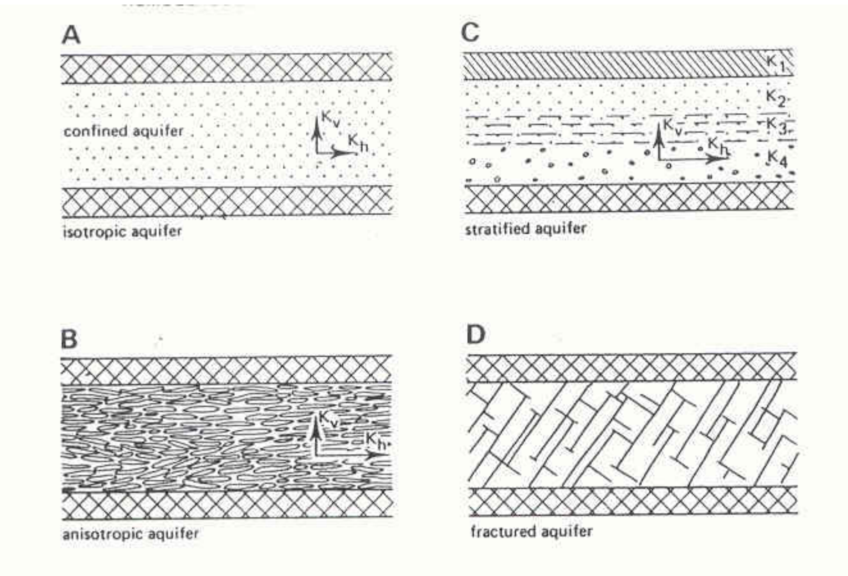

Figure 2.4 Homogeneous and heterogeneous aquifers, isotropic and anisotropic. **A.** Homogeneous aquifer, isotropic. **B.** Homogeneous aquifer, anisotropic. **C.** Heterogeneous aquifer, anisotropic, stratified, **D.** Heterogeneous aquifer, anisotropic, fractured

2.3 Compressibility and Effective Stress

2.3.1 Introduction

➢ The analysis of transient groundwater flow requires the introduction of the concept of **compressibility.**

- **Compressibility** is a material property that describes the change in volume, or strain, induced in a material under an applied stress.

- In the classical approach to the strength of elastic materials, the **modulus of elasticity** is a more familiar material property. It is defined as the ratio of the change in stress $d\sigma$ to the resulting change in the strain $d\varepsilon$ **.**

- Compressibility is simply the **inverse** of the modulus of elasticity. It is defined as strain/stress, $\frac{d\varepsilon}{d\sigma}$**,** rather than stress/strain, $\frac{d\sigma}{d\varepsilon}$.

- For the flow of water through porous media, it is necessary to define two compressibility terms, one for the water and one for the porous media.

2.3.2 Compressibility of Water (Fluid)

- Compressibility of water, β can be defined as:

$$\beta = -\frac{dV_w / V_w}{dP} \qquad \textbf{(2.8)}$$

where, V_w : volume of water, P = pressure.

- The negative sign is necessary if we wish β to be a positive number.

- An increase in pressure dP leads to a decrease in the volume V_w of a given mass of water.

- $\frac{dV_m}{V_w}$ **:** Volumetric strain induced by dP **.**

- The compressibility β is the slope of the line relating strain to stress for water, and this slope doesn't change over the range of fluid pressures encountered in groundwater hydrology (including those less than atmospheric that are encountered in the saturated zone).

- The dimensions of β are the inverse of those for pressure or stress. (m^2/N , Pa^{-1})

- **Note that :** Volume = Mass/Density, hence,

$$\beta = \frac{d\rho \;/\; \rho}{dP} \qquad \textbf{(2.9)}$$

- β ***=0****,* $\rho = \rho_o$ for an incompressible fluid. ρ_o is the fluid density at the datum pressure.

2.3.3 Effective Stress

- Let us now consider the compressibility of the porous medium. Assume that a stress is applied to a unit of saturated sand. There are three mechanisms by which a reduction in volume can be achieved:

  1. Compression of the water in the pores.
  2. Compression of the individual sand grains.
  3. Rearrangement of the sand grains into a more closely packed configuration.

The first of these mechanisms is controlled by the fluid compressibility β**.** Let us assume that the second mechanism is negligible, that is, the individual soil grains are incompressible. Our task is to define a compressibility term that will reflect the third mechanism.

- To do so**, Figure 2.5** illustrates the stresses on an arbitrary plane through a saturated porous medium.

- σ_T **:** is the total stress due to weight of overlying rock and water.

- There is an upward stress caused by fluid pressure and the actual stress that is borne by aquifer skeleton. The portion of the total stress that is not borne by fluid (i.e. borne by aquifer skeleton) is the effective stress σ_e.

- Rearrangement of soil grains and the resulting compression of the granular skeleton is caused by changes in the effective stress, not by the changes in the total stress.

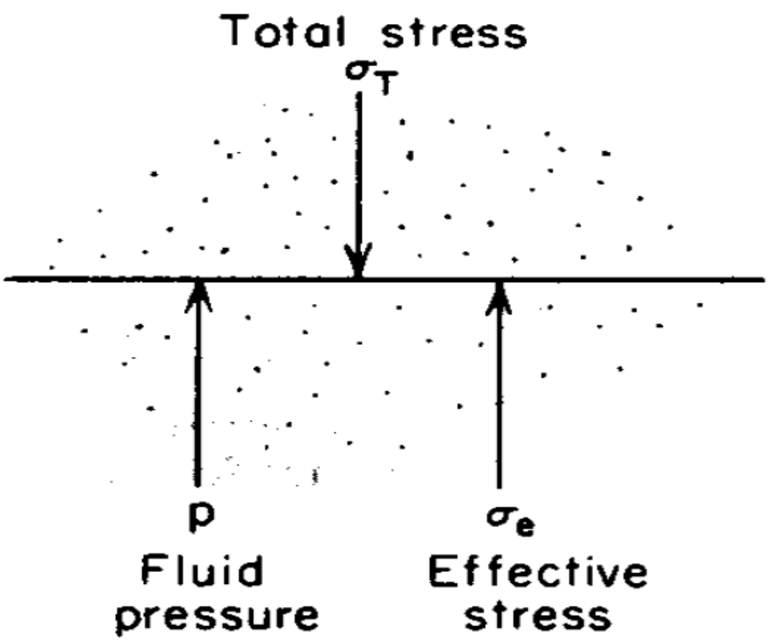

Figure 2.5 Total stress, effective stress, and fluid pressure on an arbitrary plane through a saturated porous medium

- Total stress, effective stress, and fluid pressure are related by the simple equation:

$$\sigma_T = \sigma_e + P \qquad \textbf{(2.10)}$$

or, in terms of the changes,

$$d\sigma_T = d\,\sigma_e + dP \qquad \textbf{(2.11)}$$

- The weight of the rock and water overlying each point in the system often remains essentially constant through time

$$d\sigma_T = 0 \quad \Rightarrow \quad d\sigma_e = -d\,P \qquad \textbf{(2.12)}$$

- If the fluid pressure increases, σ_e decreases by equal amount.
- If the fluid pressure decreases, σ_e increases by equal amount.
- When a well in an aquifer is being pumped, then:

  - Fluid pressure decreases, and so σ_e increases by equal amount.
  - Aquifer skeleton may compact.
  - From definition of β, Volume of water will expand

2.3.4 Aquifer Compressibility

- The compressibility of a porous medium is defined as:

$$\alpha = -\frac{dV_T \;/ V_T}{d\,\sigma_e} \qquad \textbf{(2.13)}$$

- V_T is the total volume of soil mass. $V_T = V_s + V_v$. where, V_s is the volume of the solids and V_v is the volume of the water saturated voids.

- The aquifer compressibility (see **Figure 2.6**) can be defined as:

$$\alpha = -\frac{db \;/ b}{d\,\sigma_e} \qquad \textbf{(2.14)}$$

where, - db : change in aquifer thickness.
- the negative sine indicates that the aquifer gets smaller with the increase in effective stress.

- Since, $d\,\sigma_e = -dP \Rightarrow \alpha = \dfrac{db \;/ b}{dP}$, When a well in an aquifer is being pumped (see **Figure 2.6**), then:

  - Fluid pressure decreases, and so σ_e increases by equal amount. Aquifer will be compact by db.
  - The fluid pressure increases, σ_e decreases by equal amount. **Aquifer expands.**

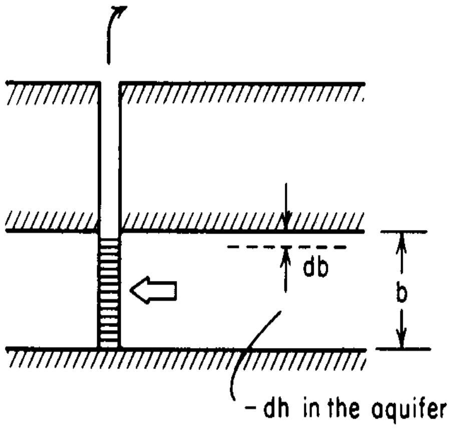

Figure 2.6 Aquifer compaction caused by groundwater pumping

- **Table 2.1** shows typical values of α which are given by Freeze and Cherry for a variety of rocks.

Table 2.1 typical values of α

	α	$m^2\ N^{-1}$
✓	Clay	$10^{-8} - 10^{-6}$
✓	sand	$10^{-9} - 10^{-7}$
✓	gravel	$10^{-10} - 10^{-8}$
✓	jointed rock	$10^{-10} - 10^{-8}$
✓	sound rock	$10^{-11} - 10^{-9}$