

2.13 HYDRAULIC CONDUCTIVITY OF FRACTURED ROCK

Joints are the most abundant structural element in the crust of the earth, and along with fractures, occur in almost all rocks near the earth's surface. Joints act as adjustment structures for the size or shape change in a rock during movement. Movement can occur due to subsidence, uplift, contraction (cooling events), expansion (unloading events), faulting and folding. Studies have shown that joints form readily in the presence of fluid: oil and water in the upper crust; magma in the lower crust.

Joints, faults and fractures serve as conduits for ground-water plumbing systems in many areas. They are often the only routes by which ground water can move through igneous and metamorphic rocks, such as granite, basalt, gabbro, quartzite, gneiss and schist. Primary porosity in these rocks is less than 1%. They can also dominate as conduits in well-cemented sedimentary rocks or act as spacings where non-migratory fluids may be trapped. Permeability in crystalline rocks range from 10^{-13} to 10^{-8} cm/s; whereas, secondary porosity can be greater than 1%. Secondary porosity and permeability decrease with depth in crystalline rocks; therefore, ground-water yield decreases. This is due to the closing of fractures with depth as overburden pressure increases. Most discontinuities close at depths ranging from 300 to 500 m (1000 to 1600 ft). Sheeting or exfoliation that occurs in near-surface crystalline rock, such as granite, can be an important source of ground water down to depths of 100 m (LeGrande, 1949).

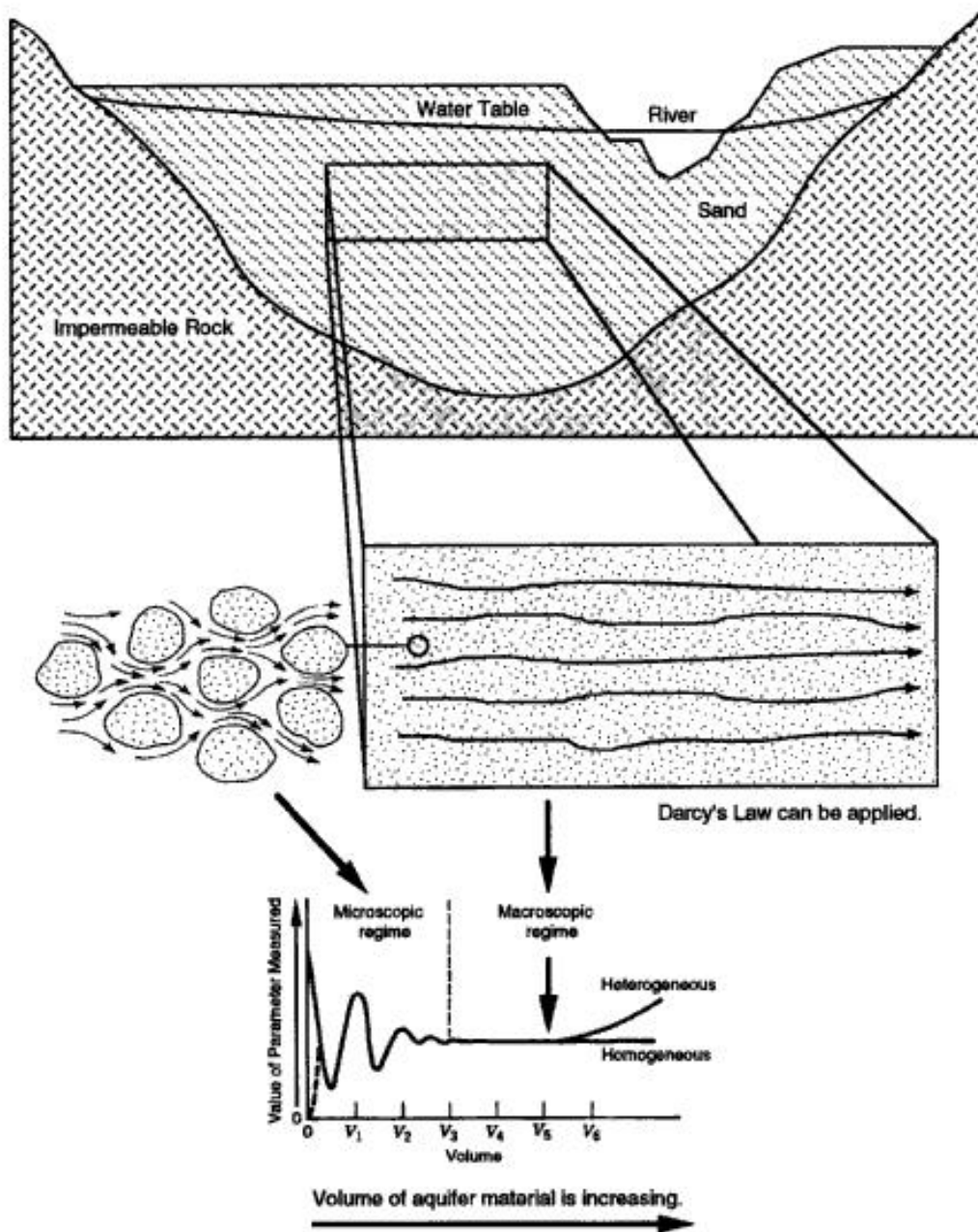


Figure 6.9. Microscopic and macroscopic domains. Notice that as the sample size increases so may heterogeneity.

Fractured rock can be treated with Darcian theory if a representative elementary volume can be identified. Identifying a representative elementary volume is often more challenging for fractured rock, because this statistical average can exist on many scales. Figure 6.10 shows how this can be applied to outcrops of basalt. "A" is the domain of a single fracture, which is analogous to the microscopic domain of a porous medium. "B" represents the statistical average of multiple fractures. "C" represents an increase in variability due to the presence of flow across the top of a bedding plane. "D" again is a representative elementary volume or statistical average, because it includes multiple flow types within the system. As with most porous systems, heterogeneity most likely increases with increases in sample size.

Snow (1968) developed equation (6.16) for the relationship between hydraulic conductivity and fractured rock. Equation (6.16) utilizes Darcian logic and it must be applied to a block of rock large enough where a representative volume can be identified. Snow's equation shows that for a given hydraulic conductivity, ground-water flow through a fracture is proportional to the cube of the fracture aperture. This means that significant increases in ground-water flow occur as the openings in fractures increase. Gale et al. (1985) point out that the roughness of a fracture or joint must be considered in regard to K. As roughness increases K decreases.

$$K = \frac{\rho_w g N b^3}{12\mu} \quad (6.16)$$

$$k = \frac{N b^3}{12} \quad (6.17)$$

where, for consistent units,

- k = permeability = L / T
- K = hydraulic conductivity = L / T
- ρ_w = density of water = M / L³
- g = acceleration due to gravity = L / T²
- N = number of joints per unit distance across the face of the rock = 1 / L
- b = mean joint aperture = L
- μ = dynamic viscosity of water = M / LT.

If Darcian flow theory cannot be applied to a fractured rock domain, then in order to quantify ground-water supply, it must be described through single fractures, joints or sets of joints and fractures. Problems beyond turbulent flow exist when considering this type of fractured rock analysis. In order to characterize flow with any professional degree of accuracy, detailed information is required regarding fracture or joint orientation, openings and connectivity. In addition, hydraulic conductivity will vary with changes in the three-dimensional fluid pressure and the three-dimensional stress field.

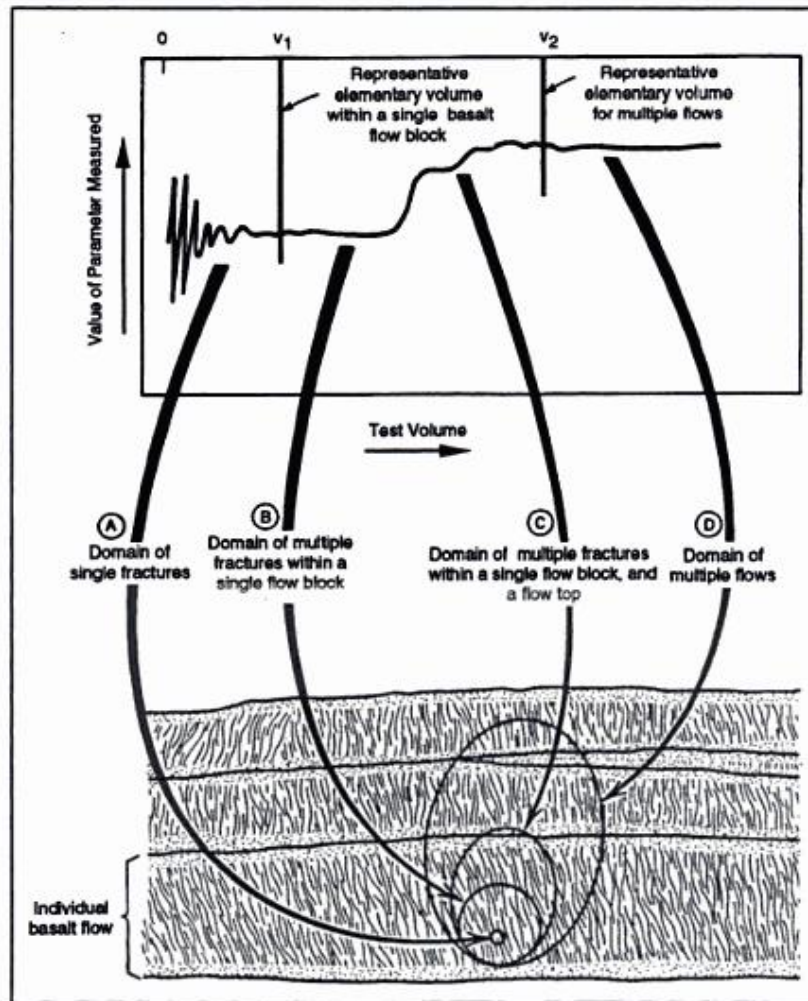


Figure 6.10. Representative elementary volume as applied to basalt (modified from Department of Energy, 1986).

As a final note: it is true that **clays and shales** often act as confining units. Hydraulic conductivities of clays measured in laboratories can be as low as 10^{-7} cm / s. However, values measured in the field may be 5 to 10,000 times greater than laboratory results (Daniel, 1984). This significant increase is often due to joints, fractures, worm holes, the work of plant roots, and desiccation cracks where environments are characterized by alternating wet and dry cycles that result in earth movements. Hasan and Hoyt (1992) recorded that desiccation cracks penetrated down to depths of 19.1 cm in a laboratory study. Fissures, fractures and joints can penetrate

much deeper in natural environments. Figure 6.11 shows an aquifer system located in the Paducah area in Kentucky. Ground water from the Holly Springs aquifer easily migrates down to the Ripley Formation due to sandstone dikes that are a few inches thick. These dikes have developed from the intrusion of sand into fractures and joints opened up by earth movements. It is clear that a large quantity of ground water leaks from one aquifer to the other, even though a thick body of Porters Creek Clay separates these units. The point is this: although clays and shales are considered to be confining units, they can transmit large quantities of ground water where discontinuities develop.

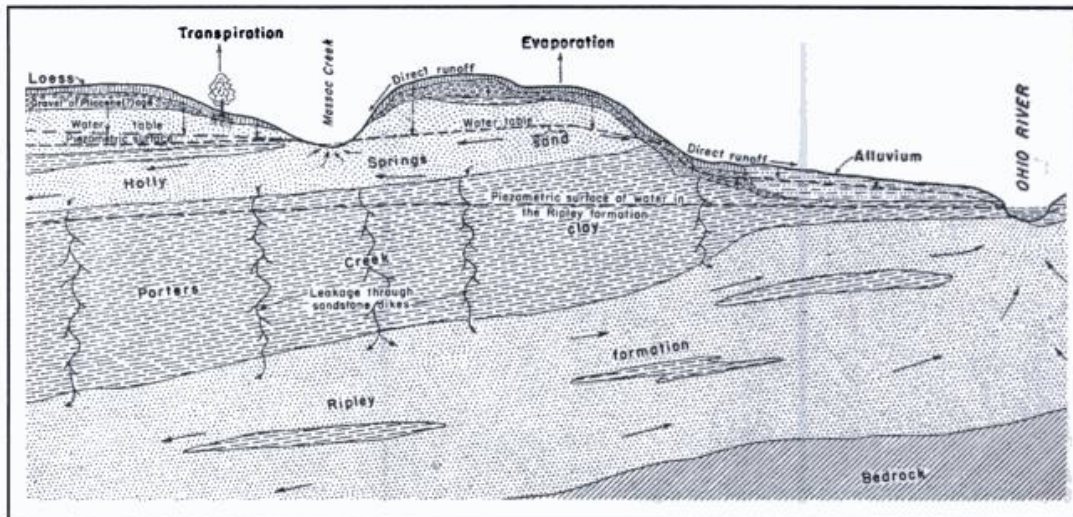


Figure 6.11. Ground-water system in the Paducah area, Kentucky. Leakage from the Holly Springs aquifer down to the Ripley Formation occurs due to the presence of sandstone dikes (source: USGS).

2.14 SATURATED FLOW AND DISPERSION

An understanding of the mechanical movement of ground water is important in regard to the movement of contaminants. Ground water and solutes that travel with ground water move by three basic mechanisms: advection, mechanical dispersion and molecular diffusion. Together, with a factor called dispersivity, this movement is called hydrodynamic dispersion.

Advection (v) is the average linear velocity of ground water, which can be determined from Darcy's Law. This is simply the velocity of ground water as demonstrated in Section 6.10:

$$v = \frac{K\Delta h}{nL} \quad 6.13$$

Example 6.8

Assume that a spill has occurred consisting of a conservative substance, such as chloride. The waste percolates down to the water table and is detected in Well 1, which has a head value equal to 825 m above mean sea level. Down gradient is Well 2, which has a head value of 815 m. The distance between Wells 1 and 2 is 2,500 m. The hydraulic conductivity of the sand and gravel aquifer is 82 m / day and the porosity is 0.25. What is the velocity of the ground water and contaminant in ft / day? How long would it take for the contaminant to travel from Well 1 to Well 2 by advection only?

First: Determine advection or the average linear velocity of ground water.

$$v = \frac{Ki}{n} = \frac{K\Delta h}{nL} \quad 6.13$$

$$\begin{aligned} K &= 82 \text{ m / day} \\ \Delta h &= (825 \text{ m}) - (815 \text{ m}) = 10 \text{ m} \\ i &= \Delta h / L = 10 \text{ m} / 2500 \text{ m} = 0.004 \\ L &= 2500 \text{ m} \\ n &= 0.25 \end{aligned}$$

$$v = \frac{Ki}{n} = \frac{\left(82 \frac{\text{m}}{\text{day}}\right)(0.004)}{(0.25)} = 1.31 \text{ m / day}.$$

Next: Determine the time it would take for the contaminant to travel the specified distance by advection only.

$$\text{Time of travel} = \frac{\text{distance}}{\text{velocity}} = \frac{2500 \text{ m}}{1.31 \frac{\text{m}}{\text{day}}} = 1,908 \text{ days} = 5.2 \text{ years}.$$

Hydrodynamic dispersion (D_x) is the mixing of ground water in both the longitudinal and transverse direction. It is dependent on **molecular diffusion (D^*)** and **mechanical dispersion ($\alpha_i v$)**. Mechanical dispersion is the product of **advection (v_x)** and a factor called **dynamic dispersivity (α_i)**.

$$D_x = \alpha_i v + D^* = L^2 / T \quad (6.18)$$

where, for consistent units,

$$\begin{aligned} D_x &= \text{longitudinal or transverse mechanical dispersion} = L^2 / T \\ \alpha_i &= \text{dynamic dispersivity} = L \\ v &= \text{advection} = L / T \\ D^* &= \text{molecular diffusion} = L^2 / T. \end{aligned}$$

Mechanical dispersion occurs, because ground water streamlines converge in the narrow pathways between sediment, and diverge as the pathways enlarge (Fig. 6.12). Differences in velocity result from friction due to various changes in pore size, the length of the flow path, and the movement of water between sediment. Ground water flows faster through large pores; slower through small pores. Some of the pathways of flow will be long; some short. As ground water moves between the walls of sediment, streamlines adjacent to the walls will move slower when compared to streamlines in the center, which move the fastest (Fig. 6.13). Dynamic dispersivity (Anderson, 1979, Neuman, 1990) is a characteristic of the porous medium and is a function of time, distance and scale (area over which it is measured). It is dependent on grain size and grain size distribution (sorting), and it is independent of grain shape, roughness and angularity. It is relatively constant at low concentrations in laboratory experiments using homogeneous material. As a rule-of-thumb dispersivity can be roughly estimated using the following equation:

$$\alpha_L = (0.1)(L) \quad (6.19)$$

where

$$\begin{aligned} \alpha_L &= \text{dispersivity in the longitudinal direction} \\ 0.1 &= 1/10\text{th of the flow path} \\ L &= \text{flow path} = L. \end{aligned}$$

According to Neuman (1990): $\alpha_L = 0.0175L^{1.46}$ (6.20)

where, for consistent units,

$$\begin{aligned} \alpha_L &= \text{dispersivity in the longitudinal direction when } L < 3500 \text{ m} \\ L &= \text{length of the flow path.} \end{aligned}$$

Molecular diffusion (D^*) is the movement of dissolved substances from high concentration to low concentration. This movement can be described by Fick's Laws of Diffusion. Fick's second law states: Dissolved ionic and molecular substances move from areas of high concentration to areas of low concentration over time --- much like smoke from a fire on a windless day dissipates into the atmosphere. Rates of diffusion for common non-reactive species in ground water range as follows (Robinson and Stokes, 1965; Freeze and Cherry, 1979):

$$\begin{aligned} \text{Fine grained clayey material: } &1 \times 10^{-10} \text{ to } 1 \times 10^{-11} \text{ m}^2/\text{s} \\ \text{Coarse grained unconsolidated material: } &1 \times 10^{-9} \text{ to } 2 \times 10^{-9} \text{ m}^2/\text{s}. \end{aligned}$$

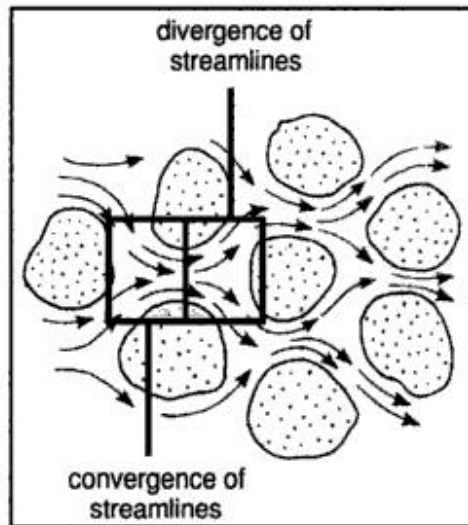


Figure 6.12. Dispersion in a granular deposit (modified from Heath, 1983).

What does all of this mean?

- 1) The bulk concentration of a contaminant or dye injected into a ground-water flow system will at first arrive at a point downstream at low concentrations as the faster streamlines arrive first.
- 2) Therefore, the concentration of a dissolved substance will decrease with distance from the source.
- 3) The front of dissolved material will move faster than would be predicted by the average linear ground-water velocity.
- 4) The concentration at the point downstream will increase until the concentration reaches about 70% of the concentration injected at the point upstream (Danel, 1953). The rate of increase in concentration will then begin to decrease.

Hydrodynamic dispersion (D_L) in the longitudinal direction is controlled by mechanical dispersion. Hydrodynamic dispersion in the transverse direction (D_T) is diffusion controlled. At high ground-water velocity, hydrodynamic dispersion in the longitudinal direction is dominate. Diffusion (D^*) can be neglected and the hydrodynamic dispersion equation can be expressed as (Fig. 6.14)

$$D_L = \alpha_L v. \quad (6.21)$$

The plume will be elongated in the direction of ground-water flow when $D_L > D_T$. This is the case for ground water flowing through homogeneous sand or gravel. At low ground-water velocity, mechanical dispersion ($\alpha_L v$) can be neglected and hydrodynamic dispersion expressed as (Fig. 6.15).

$$D_L = D^*. \quad (6.22)$$

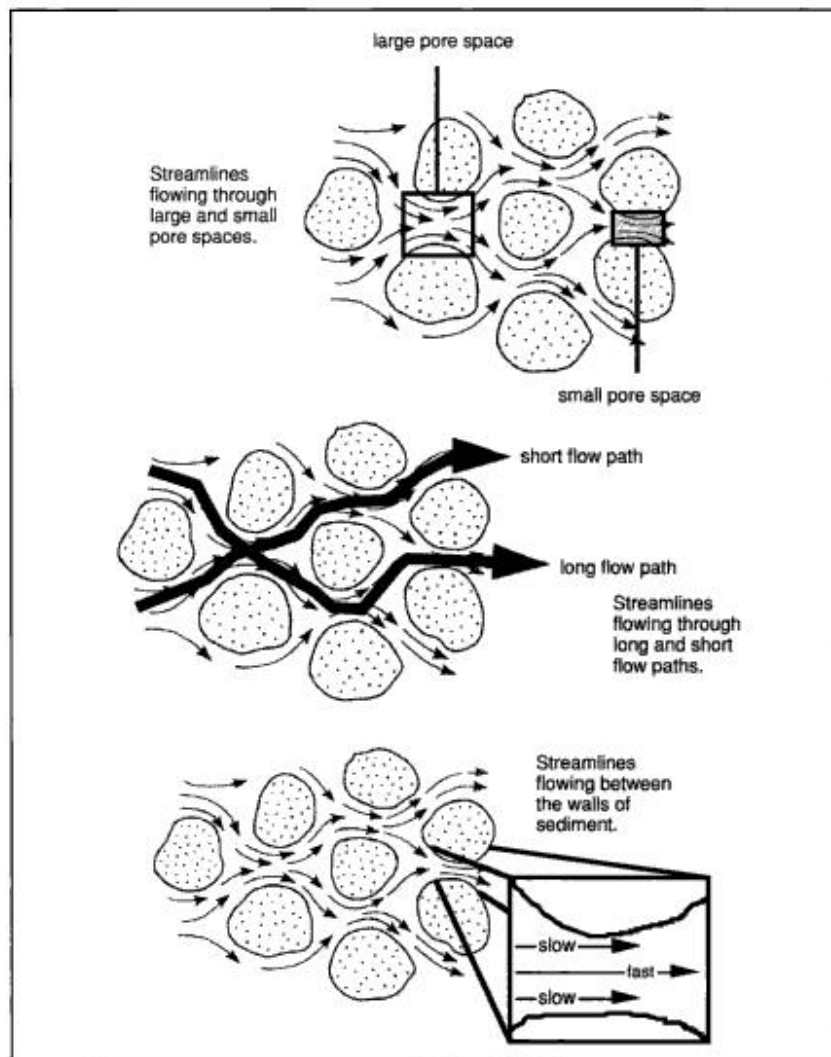


Figure 6.13. Factors that determine dispersion in granular material.

The contaminant plume will tend to be spherical when $D_L \cong D_T$. This is the case for ground water flowing through silty or clayey material.

As Count Von Count once said to Ernie: "It's not as easy as it seems."

Many factors must be considered in regard to contaminant analysis of aquifer systems. They include, but are not restricted to:

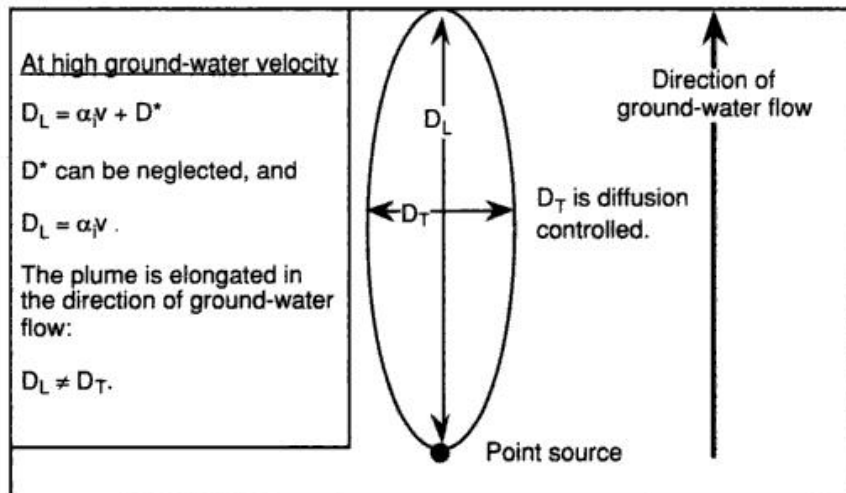


Figure 6.14. Geometry of a contaminant plume moving through homogeneous coarse grained material.

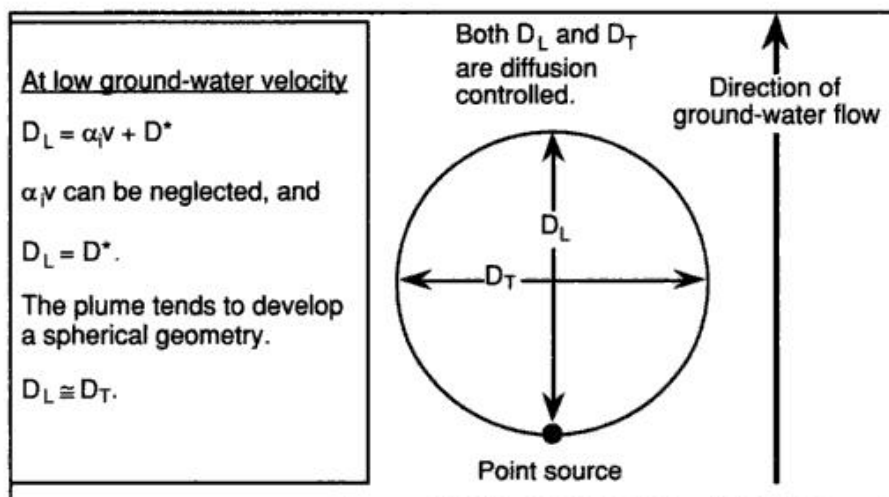


Figure 6.15. Geometry of a contaminant plume moving through fine grained material.

I) Aquifer Parameters

- 1) Transmissivity
- 2) Hydraulic conductivity
- 3) Storage coefficient
- 4) Porosity
- 5) Ground-water velocity

II) Source

- 1) Point source: known area or source where contaminant is introduced.
- 2) Non point source: source of contaminant is unknown or cannot be determined.
- 3) One time source: introduction of a contaminant at a specified time and location, as when a spill occurs.
- 4) Continuous source: contaminant flows into the environment at some rate over a lengthy time period.

III) Chemical Processes

- 1) Chemical composition of the contaminant.
 - 2) pH of solution.
 - 3) Retardation: a chemical process that tends to remove contaminants from ground water (below is a common list).
- Adsorption/Desorption: ions adhering or being removed from the surfaces of clay.
 - Cation Exchange: cations being removed and replaced by more preferred cations.
 - Precipitation/Dissolution: solids precipitating from or dissolving into solution.
 - Oxidation and Reduction: molecular or ionic loss or gain of electrons.
 - Microbial Decay: complex compounds decaying into simpler compounds by the work of bacteria. The more complex compounds are often non-carcinogens, while the simpler compounds are often carcinogens.

IV) Geology

- 1) Geologic material (clay and silt vs sand and gravel).
- 2) Sorting of material.
- 3) Faults / fractures / joints.
- 4) Thickness of units.

- 5) Mineralogy.
- 7) Stratigraphy.
- 8) Geologic structures.
- 9) Environment (reducing vs oxidizing).
- 10) Complexity of the ground-water system.

2.15 GROUND-WATER CONTAMINATION

About half of us living in the United States depend upon ground-water for potable water. Obviously the problem must be dealt with. This hazard is dependent upon many factors, including, but not limited to, volume of pollutant introduced into the aquifer, concentration of the contaminant, and the degree of exposure to living organisms. Good news and bad news is in the offering when considering ground-water contamination. In regard to good news, you should now be aware that the movement of ground-water through an aquifer system is generally slow. Exceptions do exist, such as in some karst and volcanic terranes, but in general, where contaminants are known to migrate, time is an ally, allowing clean-up strategies to be formulated without the human desire to panic. The dilution and dispersion of a contaminant will be limited and contained within a plume. Now for the bad news. Ground-water systems and chemical reactions are complex. Predictability, in regard to the movement of contaminant plumes, is therefore treated with many disclaimers. In addition, where contaminated ground water is discharged into streams and rivers up gradient, systems down gradient may also become polluted, because the same stream will flow into a lake or wetland, or may be a tributary for another system. Cost is another problem when considering ground-water clean-up. Billions of dollars are spent each year on legal and technical fees, which of course supports the obvious: it is cheaper to prevent contamination than to clean it up once it occurs.

MASS TRANSPORT OF SOLUTES ✓

11.6.1 Introduction

In studying ground-water contamination it is helpful to understand the basic theory behind the movement of solutes contained in ground water. In the study of water chemistry, the processes by which substances can become dissolved in water are examined. However, the processes by which these substances move through porous media are complex. They can be expressed mathematically, although in some instances we do not fully understand how to obtain the field data necessary to apply the theoretical equations.

There are two basic processes operating to transport solutes. **Diffusion** is the process by which both ionic and molecular species dissolved in water move from areas of higher concentration (i.e., chemical activity) to areas of lower concentration. **Advection** is the process by which moving ground water carries with it dissolved solutes. We will see how, as solutes are carried through porous media, the process of **dispersion** acts to dilute the solute and lower its concentration. Finally, there are chemical and physical processes that cause **retardation** of solute movement so that it may not move as fast as the advection rate would indicate.

11.6.2 Diffusion

The diffusion of a solute through water is described by Fick's laws. Fick's first law describes the flux of a solute under steady-state conditions:

$$F = -D \, dC/dx \quad (11-1)$$

where

F = mass flux of solute per unit area per unit time

D = diffusion coefficient (area/time)

C = solute concentration (mass/volume)

dC/dx = concentration gradient (mass/volume/distance)

The negative sign indicates that the movement is from greater to lesser concentrations. Values for D are well known for electrolytes in water. For the major cations and anions in water, D ranges from 1×10^{-9} to 2×10^{-9} m²/s.

For systems where the concentrations may be changing with time, Fick's second law may be applied:

$$\partial C / \partial t = D \partial^2 C / \partial x^2 \quad (11-2)$$

where $\partial C / \partial t$ = change in concentration with time.

Both Fick's first and second law as expressed above are for one-dimensional situations. For three-dimensional analysis, more general forms would be needed.

In porous media, diffusion cannot proceed as fast as it can in water because the ions must follow longer pathways as they travel around mineral grains. In addition, the diffusion can take place only through pore openings because mineral grains block many of the possible pathways. To take this into account, an effective diffusion coefficient must be used. This is termed D^* .

The value of D^* can be determined from the relationship

$$D^* = wD \quad (11-3)$$

where w is an empirical coefficient that is determined by laboratory experiments. For species that are not adsorbed onto the mineral surface it has been determined that w ranges from 0.5 to 0.01 (Freeze & Cherry 1979). Berner (1971) gave a nonempirical relationship between D^* and D that indicated that D^* was equal to D times the porosity divided by the square of the **tortuosity** of the flow path of the diffused species. Tortuosity is the actual length of the flow path, which is sinuous in form, divided by the straight-line distance between the ends of the flow path. Unfortunately, tortuosity cannot be determined in the field, and one is left with the experimental approach.

The process of diffusion is complicated by the fact that ions must maintain electrical neutrality as they diffuse. If we have a solution of NaCl, the Na^+ cannot diffuse faster than the Cl^- , unless there is some other negative ion in the region into which the Na^+ is diffusing.

It should also be mentioned at this point that if the solute is adsorbed onto the mineral surfaces of the porous medium, the net rate of diffusion will obviously be less than for a nonadsorbed species. This topic is addressed more fully in the section on retardation.

It is possible for solutes to move through a porous medium by diffusion, even though the ground water is not flowing. Thus, even if the hydraulic gradient is zero, a solute could still move. In rock and soil with very low permeability, the water may be moving very slowly. Under these conditions, diffusion might cause a solute to travel faster than the ground water is flowing. Under such conditions, diffusion is more important than advection.

11.6.3 Advection

The rate of flowing ground water can be determined from Darcy's law as

$$v_x = \frac{K}{n_e} \frac{dh}{dl} \quad (11-4)$$

where

v_x = average linear velocity

K = hydraulic conductivity

n_e = effective porosity

$\frac{dh}{dl}$ = hydraulic gradient

Contaminants that are advecting are traveling at the same rate as the average linear velocity of the ground water.

11.6.4 Mechanical Dispersion

As a contaminated fluid flows through a porous medium, it will mix with noncontaminated water. The result will be a dilution of the contaminant by a process known as dispersion. The mixing that occurs along the streamline of fluid flow is called longitudinal dispersion. Dispersion that occurs normal to the pathway of fluid flow is lateral dispersion.

There are three basic causes of pore-scale longitudinal dispersion: (1) As fluid moves through pores, it will move faster through the center of the pore than along the edges. (2) Some of the fluid will travel in longer pathways than other fluid. (3) Fluid that travels through larger pores will travel faster than fluid moving in smaller pores. This is illustrated by Figure 11.8.

Lateral dispersion is caused by the fact that, as a fluid containing a contaminant flows through a porous medium, the flow paths can split and branch out to the side. This will occur even in the laminar flow conditions that are prevalent in ground-water flow (Figure 11.9).

The mechanical dispersion due to the preceding factors is equal to the product of the average linear velocity and a factor called the dynamic dispersivity (a_L).

$$\text{Mechanical dispersion} = a_L v_x \quad (11-5)$$

11.6.5 Hydrodynamic Dispersion

The processes of molecular diffusion and mechanical dispersivity cannot be separated in flowing ground water. Instead, a factor termed the coefficient of hydrodynamic dispersion, D_L , is introduced. It takes into account both the mechanical mixing and diffusion. For one-dimensional flow it is represented by the following equation:

$$D_L = a_L v_x + D^* \quad (11-6)$$

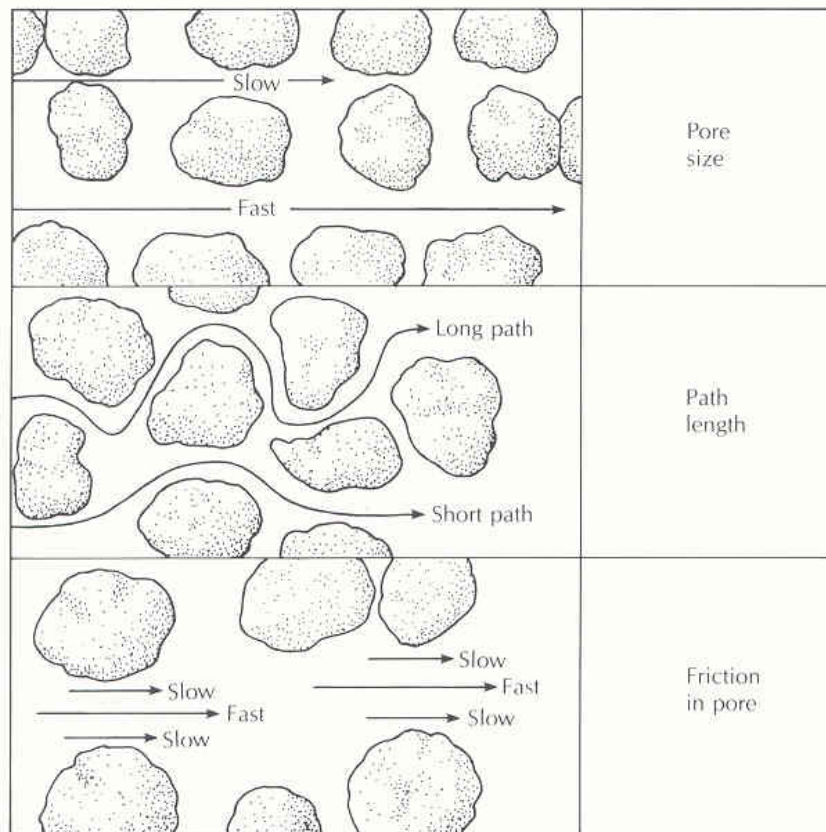


FIGURE 11.8 Factors causing pore-scale longitudinal dispersion.

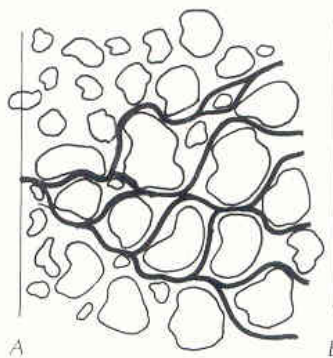


FIGURE 11.9 Flow paths in a porous medium that cause lateral hydrodynamic dispersion.

where

D_L = the longitudinal coefficient of hydrodynamic dispersion

a_L = the dynamic dispersivity

v_x = the average linear ground-water velocity

D^* = the molecular diffusion

The process of longitudinal hydrodynamic dispersion can be illustrated by the following simple experiment. A tube is filled with sand and set up so that distilled water is flowing through it at a constant rate. We then change the influent to a 1% saline solution and begin to monitor the effluent for chloride. The effluent has zero chloride initially, as distilled water is still flushing from the tube. Eventually we will begin to detect chloride in the effluent water. It arrives initially at a very low concentration. The “breakthrough” is not at the 1% concentration. This small amount gradually increases until the 1% saline concentration is reached. The first chloride ions to arrive traveled through the shortest flow paths. Diffusion in advance of the advecting water may have even caused some of the chloride to reach the outlet prior to water that was advecting it. The initial chloride was being diluted by the distilled water that was arriving at the same time. The amount of distilled water available for dilution continually decreased until the saline solution filled all the pores and the effluent water was at the influent concentration. Figure 11.10 illustrates this process.

The one-dimensional equation for hydrodynamic dispersion (Beruch & Street 1967; Hoopes & Harleman 1967) is given by

$$D_L \frac{\partial^2 C}{\partial x^2} - v_x \frac{\partial C}{\partial x} = \frac{\partial C}{\partial t} \quad (11-7)$$

where

D_L is the coefficient of longitudinal hydrodynamic dispersion

C is the solute concentration

v_x is the average ground-water velocity in the x -direction

t is the time since start of solute invasion

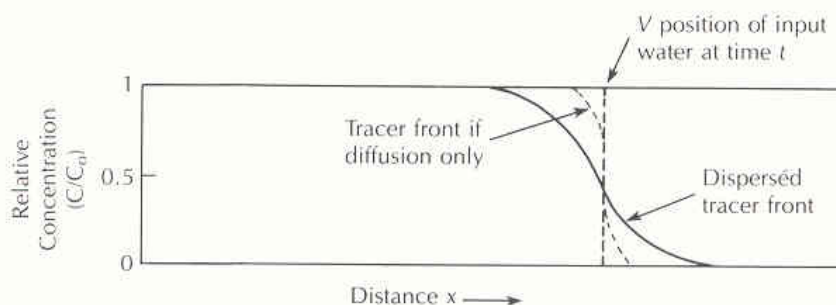


FIGURE 11.10 Influence of dispersion and diffusion on “breakthrough” of a solute.

The concentration, C , at some distance, L , from the source at concentration, C_0 , at time, t , is given by the following expression (Ogata 1970), where erfc is the complementary error function:

$$C = \frac{C_0}{2} \left[\text{erfc}\left(\frac{L - v_x t}{2\sqrt{D_L t}}\right) + \exp\left(\frac{v_x L}{D_L}\right) \text{erfc}\left(\frac{L + v_x t}{2\sqrt{D_L t}}\right) \right] \quad (11-8)$$

where

- C is the solute concentration (M/L^3 , mg/L)
- C_0 is the initial solute concentration (M/L^3 , mg/L)
- L is the flow path length (L ; ft or m)
- v_x is the average linear ground water velocity (L/T ; ft/day or m/day)
- t is the time since release of the solute (T ; day)
- D_L is the longitudinal dispersion coefficient (L ; ft or m)

The advection-dispersion equation is based on the premise that the center of mass of the solute is moving at the same rate as the average linear ground-water velocity. Furthermore it is assumed that hydrodynamic dispersion causes the solute to spread out both ahead of and behind the center of mass in a pattern that follows a statistically normal distribution. This is the familiar bell-shaped curve. The normal distribution is also sometimes called a Gaussian distribution. Figure 11.11 shows the movement of a solute mass and the way that it spreads with time and distance. The solute front is moving at a rate that is greater than would be predicted by the average linear ground-water velocity.

Mechanical dispersion is also caused by the heterogeneities in the aquifer. As ground-water flow proceeds in an aquifer, regions of greater than average hydraulic conductivity and lesser than average conductivity will be encountered. The resulting variation in linear ground-water velocity results in much greater hydrodynamic dispersion than that caused by the pore-scale effects. The greater

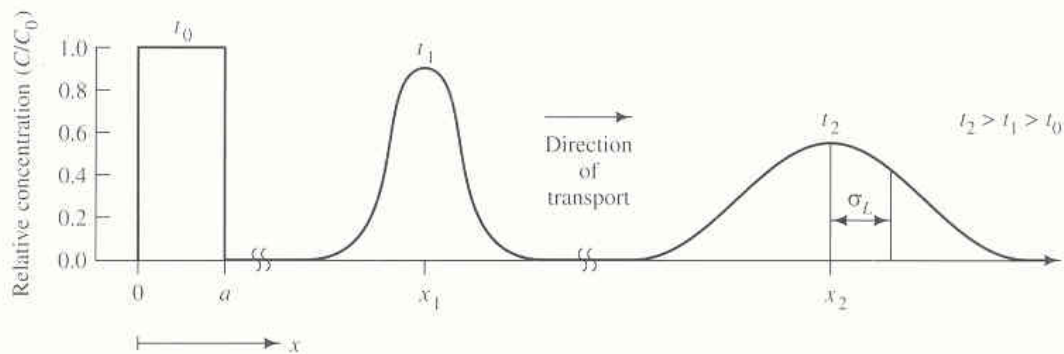


FIGURE 11.11 Transport and spreading of a solute slug with time due to advection and dispersion. A slug of solute was injected at $x = 0 + a$ at time t_0 with a resulting concentration of C_0 . The ground-water flow is to the right. Source: C. W. Fetter, *Contaminant Hydrogeology*. New York: Macmillan Publishing Company, 1993.

the distance over which dispersivity is measured, the greater the value that is observed. This has been called the *scale effect* (Anderson 1979). Pore-scale dispersion measured in the laboratory is on the order of centimeters, whereas macrodispersion measured in the field is on the order of meters. Neuman (1990) made a study of the relationship between the apparent longitudinal dynamic dispersivity and the flow length. He found that for flow paths less than 3500 m long, these could be related by the equation:

$$a_L = 0.0175L^{1.46} \quad (11-9)$$

where

a_L is the apparent longitudinal dynamic dispersivity (L ; ft or m)

L is the length of the flow path (L ; ft or m)

Because of hydrodynamic dispersion, the concentration of a solute will decrease with distance from the source. The solute will spread in the direction perpendicular to the flow. This is because the longitudinal dispersivity is greater than the lateral dispersivity. A continuous source will yield a plume, whereas a spill will yield a slug that grows with time as it moves down the ground-water flow path. This is illustrated by Figure 11.12.

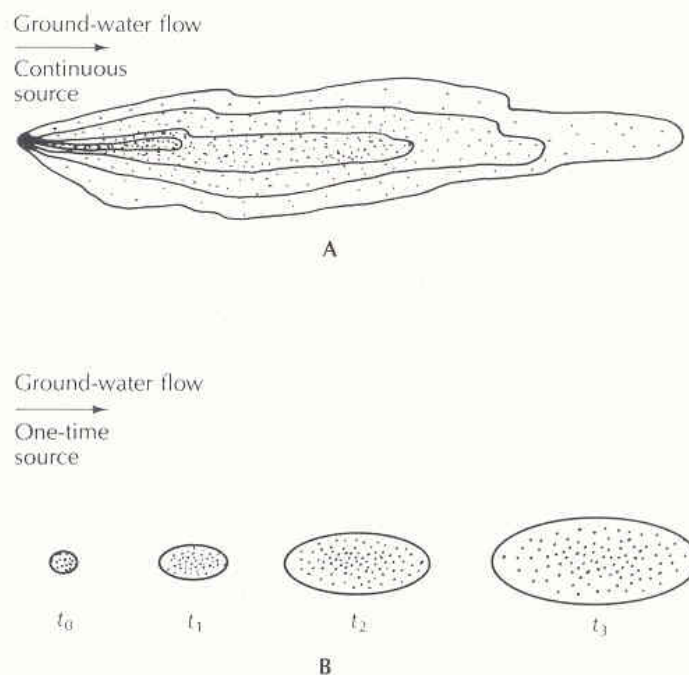


FIGURE 11.12 A. The development of a contamination plume from a continuous point source. B. The travel of a contaminant slug from a one-time point source. Density of dots indicates solute concentration.

Heterogeneities in the aquifer can cause the pattern of the solute movement to vary from what one might expect in homogeneous beds. Because flowing ground water always follows the most permeable pathways, those pathways will also have the most contaminant.

EXAMPLE PROBLEM A landfill is leaking leachate with a chloride concentration of 725 mg/L, which enters an aquifer with the following properties:

$$\text{Hydraulic conductivity} = 3.0 \times 10^{-3} \text{ cm/s } (3.0 \times 10^{-5} \text{ m/s})$$

$$dh/dl = 0.0020$$

$$\text{Effective porosity} = 0.23$$

$$D^* = 1 \times 10^{-9} \text{ m}^2/\text{s} \text{ (estimated)}$$

Compute the concentration of chloride in 1 y at a distance 15 m from the point where the leachate entered the ground water.

1. Determine average linear velocity.

$$\begin{aligned} v_x &= \frac{K(dh/dl)}{n_e} = (3.0 \times 10^{-5} \text{ m/s} \times 0.002)/(0.23) \\ &= 2.6 \times 10^{-7} \text{ m/s} \end{aligned}$$

2. Determine the coefficient of longitudinal hydrodynamic dispersion.
 - a. Find the value of a_L from Equation 11-9.

$$\begin{aligned} a_L &= 0.0175L^{1.46} \\ &= 0.0175 (15 \text{ m})^{1.46} \\ &= 0.91 \text{ m} \end{aligned}$$

- b. Find the value of D_L .

$$\begin{aligned} D_L &= a_L \times v_x + D^* \\ &= (0.91 \text{ m} \times 2.6 \times 10^{-7} \text{ m/s}) + 1 \times 10^{-9} \text{ m}^2/\text{s} \\ &= 2.4 \times 10^{-7} \text{ m}^2/\text{s} \end{aligned}$$

3. Restate the 1-y time of travel in seconds.

$$\begin{aligned} t &= 1 \text{ y} \times 60 \text{ s/min} \times 1440 \text{ min/day} \times 365 \text{ days/y} \\ &= 3.15 \times 10^7 \text{ s} \end{aligned}$$

4. Substitute values into Equation 11-8.

$$C_0 = 725 \text{ mg/L}$$

$$L = 15 \text{ m}$$

$$t = 3.15 \times 10^7 \text{ s}$$

$$D_L = 2.4 \times 10^{-7} \text{ m}^2/\text{s}$$

$$v_x = 2.6 \times 10^{-7} \text{ m/s}$$

$$\begin{aligned} C &= \frac{725}{2} \operatorname{erfc} \left(\frac{15 \text{ m} - (2.6 \times 10^{-7} \text{ m/s} \times 3.15 \times 10^7 \text{ s})}{2 \times [2.4 \times 10^{-7} \text{ m}^2/\text{s} \times 3.15 \times 10^7 \text{ s}]^{.5}} \right) \\ &\quad + \left[\exp \left(\frac{2.6 \times 10^{-7} \text{ m/s} \times 15 \text{ m}}{2.4 \times 10^{-7} \text{ m}^2/\text{s}} \right) \right. \\ &\quad \left. \times \operatorname{erfc} \left(\frac{(15 \text{ m} + (2.6 \times 10^{-7} \text{ m/s} \times 3.15 \times 10^7 \text{ s}))}{2 \times [2.4 \times 10^{-7} \text{ m}^2/\text{s} \times 3.15 \times 10^7 \text{ s}]^{.5}} \right) \right] \text{ mg/L} \\ &= 362.5 \left[\operatorname{erfc} \left(\frac{15 \text{ m} - 8.19 \text{ m}}{5.5 \text{ m}} \right) + \exp(16.25) \times \operatorname{erfc} \left(\frac{15 \text{ m} + 8.19 \text{ m}}{5.5 \text{ m}} \right) \right] \text{ mg/L} \\ &= 362.5 [\operatorname{erfc}(1.24) + \exp(16.25) \times \operatorname{erfc}(4.22)] \text{ mg/L} \end{aligned}$$

The complementary error function can be determined from Appendix 13. Since the complementary error function of numbers greater than 3 is infinitesimally small, we may ignore the second term of the equation.

$$\begin{aligned} C &= 362.5 \times 0.083 \text{ mg/L} \\ &= 30 \text{ mg/L} \end{aligned}$$

Case 1: 1D convection-dispersion: instantaneous source

- Analytical solution

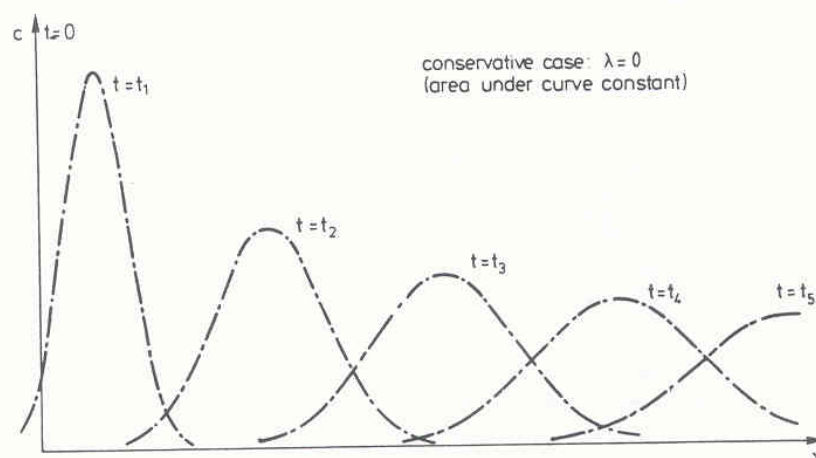
$$\frac{c}{c_0} = \frac{1}{\sqrt{4\pi D_L t}} \exp \left[-\frac{(x-Vt)^2}{4D_L t} \right]$$

is identical to Normal Distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right]$$

with

$$\mu = Vt, \quad \sigma = \sqrt{2D_L t}$$



APPENDIX 13

Values of the error function of x [$\text{erf}(x)$] and the complementary error function of x [$\text{erfc}(x)$]. Note that $\text{erfc}(x) = 1 - \text{erf}(x)$ and $\text{erfc}(-x) = 1 + \text{erf}(x)$.

x	$\text{erf}(x)$	$\text{erfc}(x)$
0	0	1.0
0.05	0.056372	0.943628
0.1	0.112463	0.887537
0.15	0.167996	0.832004
0.2	0.222703	0.777297
0.25	0.276326	0.723674
0.3	0.328627	0.671373
0.35	0.379382	0.620618
0.4	0.428392	0.571608
0.45	0.475482	0.524518
0.5	0.520500	0.479500
0.55	0.563323	0.436677
0.6	0.603856	0.396144
0.65	0.642029	0.357971
0.7	0.677801	0.322199
0.75	0.711156	0.288844
0.8	0.742101	0.257899
0.85	0.770668	0.229332
0.9	0.796908	0.203092
0.95	0.820891	0.179109
1.0	0.842701	0.157299
1.1	0.880205	0.119795
1.2	0.910314	0.089686
1.3	0.934008	0.065992
1.4	0.952285	0.047715
1.5	0.966105	0.033895
1.6	0.976348	0.023652
1.7	0.983790	0.016210
1.8	0.989091	0.010909
1.9	0.992790	0.007210
2.0	0.995322	0.004678
2.1	0.997021	0.002979
2.2	0.998137	0.001863
2.3	0.998857	0.001143
2.4	0.999311	0.000689
2.5	0.999593	0.000407
2.6	0.999764	0.000236
2.7	0.999866	0.000134
2.8	0.999925	0.000075
2.9	0.999959	0.000041
3.0	0.999978	0.000022
∞	1.00000	0.00000

An approximate solution, correct to within 0.7%, of the error function can be determined analytically from the following equation:

$$\text{erf}(x) = \sqrt{1 - \exp\left(\frac{-4x^2}{\pi}\right)}$$