SESSION # TWO

BASIC PRINCIPLES OF GROUNDWATER HYDROLOGY II

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1. Heterogeneity and Anisotropy of Hydraulic Conductivity

- <u>Heterogeneity</u> is the change of a property in space.
- <u>Anisotropy</u> is the change of a property with the direction of measurement.
- If (K) is independent of position within a geological formation, then the formation is <u>homogenous</u>.
- If (K) is dependent on position within a geological formation, then the formation is <u>heterogeneous</u>.

- **If (K) is independent of the direction of measurement, then the formation is <u>isotropic</u>.**

- **If (K) is dependent on the direction of measurement, then the formation is <u>anisotropic</u>.**

- **Note: Statistical distributions are used to provide a quantitative description of the degree of heterogeneity in a geological formation.**

- **See Figure 1**

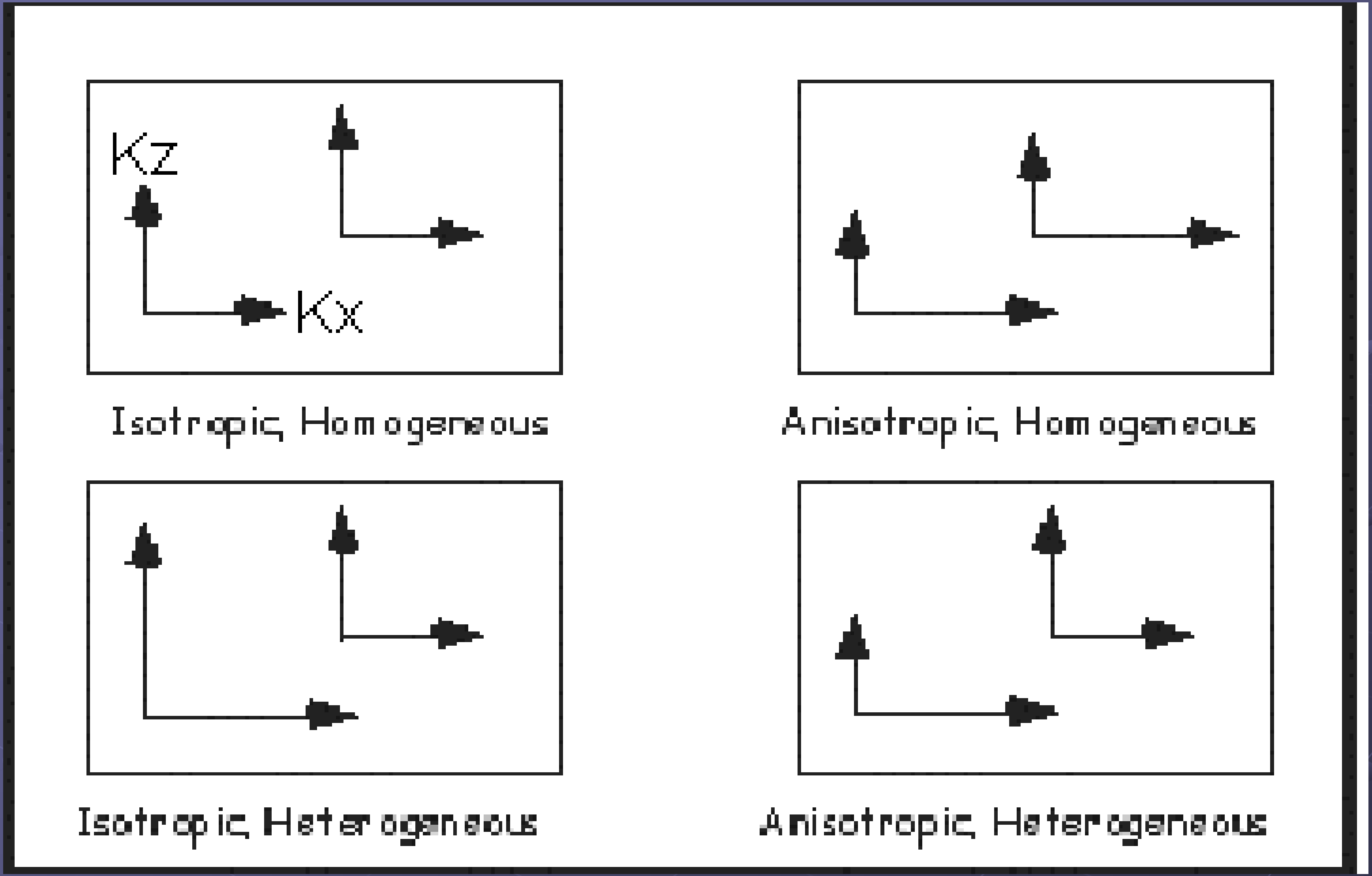


Figure 1 **Four possible combinations of heterogeneity and anisotropy**

2 Process Governing Flow in Porous Media

The steady state flow of fluid through a porous media is governed by physical processes which are expressed mathematically by two fundamental equations:

1. Darcy's Law, which expresses the relationship between the motive force applied to the fluid and the resulting discharge of fluid through the medium.

2. Continuity Equation, which expresses the conservation of fluid mass within the system.

2.1 Darcy's Law for flow porous media

- The classic work on the flow of water through a porous medium was conducted by ***Henri Darcy in France in 1856***. Darcy's result is of fundamental importance and remains at the heart of almost all groundwater flow calculations.

- Darcy discovered that the ***discharge Q of water through a column of sand is proportional to the cross sectional area A of the sand column***, and to the ***difference in piezometric head between the ends of the column***, h1– h2, and ***inversely proportional to the length of the column L***. That is:

$$Q = KA \frac{h_1 - h_2}{L}$$

- Darcy’s experiment is shown schematically in Figure 3. The constant of proportionality *K* is known as the hydraulic conductivity [LT–1] and is a measure of the ease with which water can be moved through the porous medium.

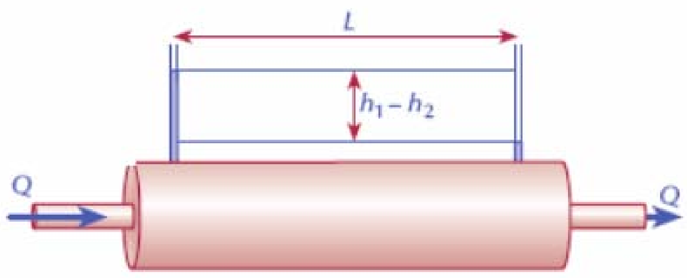


Figure 3 A schematic diagram of Darcy’s experiment

- Rather than referring to the total discharge Q, it is often more convenient to standardize the discharge by considering the volume flux of water through the column, i.e. the discharge across a unit area of the porous medium. In the context of groundwater, ***the volume flux is called the specific discharge*** q [m/sec] and is given simply by Q/A. Darcy's result can then be written in terms of the specific discharge and the difference in head between the ends of the column.

$$q = \frac{Q}{A} = -K\,\frac{h_1 - h_2}{L}$$

- The fraction (h2 – h1)/L is called the ***average hydraulic gradient*** over the length of the column. As L tends to zero, the average hydraulic gradient becomes an increasingly close approximation to the point value of the derivative of head with respect to distance *x*. Darcy's experimental result then becomes:

$$q = -K \frac{dh}{dx}$$

- which describes Darcy's Law at any point in the porous medium. The spatial derivative of head ***dh/dx*** is called the ***hydraulic gradient*** at that point.

2.2 Specific discharge and groundwater velocity

- **There is a fundamental relationship between *<u>specific discharge and groundwater velocity</u>*. Specific discharge has the dimensions of velocity, and in some books it is referred to as the Darcy velocity.**

- **This terminology is misleading and is best avoided, as *<u>the specific discharge is not a velocity</u>* – and it is certainly not the same as the groundwater velocity. To illustrate the difference, consider what happens when *<u>we pump water through an empty pipe.</u>***

- **The relationship between discharge, cross-sectional area and water velocity u is *<u>u = Q/A</u>*, and in this case the velocity *is* equal to the specific discharge.**

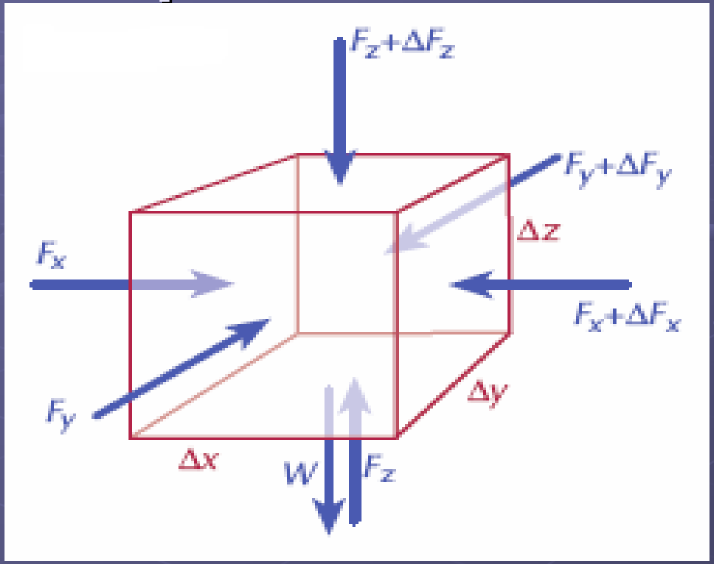
- However, if we repeat the experiment but this time fill the pipe with sand, the cross-sectional area of the pipe remains the same, but the cross sectional area that is open to flow is much reduced, and so, for the same discharge (and hence the same specific discharge) through the pipe, the water will be forced through a smaller cross-sectional area and will, therefore, have to travel faster than if the pipe were empty. This means that the water velocity will be higher than the specific discharge.

- It can be shown that the effective area open to flow is Ane, where ne is the effective porosity of the rock, and hence the groundwater velocity can be calculated by:

$$u = \frac{q}{n_e}$$

- Note that this does not mean that water travels more easily through low porosity rock. It does mean that if the specific discharge through two rocks is the same, then the water will travel faster through the rock with the lower effective porosity.

2.3 The forces causing the motion of water in a porous media



The resultant of the fluid weight and the forces due to hydrostatic pressure is given by:

$$\left(F_x - (F_x + \Delta F_x)\right)i + \left(F_y - (F_y + \Delta F_y)\right)j + \left(F_z - (F_z + \Delta F_z) - W\right)k$$

or,

$$-\Delta F_x i + \Delta F_y j - \left(\Delta F_z + W\right)k$$

where i, j and k are the unit vectors in the *x*, *y* and *z* directions respectively.

In terms of the pressures on each face of the volume, the resultant force can be written:

$$\left(P_x - (P_x + \Delta P_x)\right)\Delta_y \Delta_z i + \left(P_y - (P_y + \Delta P_y)\right)\Delta_z \Delta_x j + \left(P_z - (P_z + \Delta P_z) - \rho g \Delta_x \Delta_y \Delta_z\right)k$$

or,

$$-\Delta P_x \Delta_y \Delta_z i - \Delta P_y \Delta_z \Delta_x j - \left(\Delta P_z + \rho g \Delta z\right)\Delta_x \Delta_y k$$

Dividing by ΔxΔyΔz gives the force per unit volume

$$F = -\frac{\Delta P_x}{\Delta_x} i - \frac{\Delta P_y}{\Delta_y} j - \left(\frac{\Delta P_z}{\Delta_z} + \rho g \right) k$$

Now letting Δ*x*, Δ*y* and Δ*z* tend to zero gives,

$$F = -\frac{\partial P}{\partial x} i - \frac{\partial P}{\partial y} j - \left(\frac{\partial P}{\partial z} + \rho g \right) k$$

$$= -\nabla P - \rho g k$$

$$= -\nabla P - \rho g \nabla z$$

The above equation means that the driving force on the water is a function of the pressure distribution and the weight of the fluid

2.4 Darcy law in isotropic media (the general case)

In an isotropic medium we assume that there are no preferential directions within the medium and consequently that the specific discharge is in the same direction as the resultant force on a volume of fluid. Darcy’s Law states that the specific discharge is proportional to the head or energy gradient which represents the force applied to a volume of water and so this can be written.

$$q = -\frac{k}{\mu}\left(\nabla P + \rho g \nabla z\right)$$

$$= -\frac{k}{\mu}\rho g\left(\frac{1}{\rho g}\nabla P + \nabla z\right)$$

Notice that the intrinsic permeability is a property of the porous medium whereas the dynamic viscosity is a property of the fluid.

Notice that the expression for q above cannot be simplified to:

$$q = -\frac{k}{\mu}\rho g \nabla\left(\frac{P}{\rho g} + \nabla z\right)$$

since we cannot take the 1/ρg inside the operator. To do this we require that ρ and g be constant.

2.5 Darcy's law in isotropic porous media (NON-Constant Density)

The general form of Darcy's Law is written in terms of pressure, and this is essential if the density of the groundwater is spatially variable. However, if the density is homogeneous (i.e. constant in space), then we can make a simplification that allows it to be written, like Darcy's experimental result, in terms of head.

The acceleration due to gravity is a physical constant, but if it is assumed that ρ is also constant, then we can write:

$$q = -\frac{k}{\mu}\rho g \nabla\left(\frac{P}{\rho g} + z\right)$$

$$= -\frac{k}{\mu}\rho g \nabla h$$

$$= -K \nabla h$$

or, it can also be written in different notation as:

$$q = -K\left(\frac{\partial h}{\partial x} i + \frac{\partial h}{\partial y} j + \frac{\partial h}{\partial z} k\right)$$

Which is a simple extension of Darcy's experimental result to two or three dimensions. However, it is important to realize that this equation is valid only if the density of the fluid is constant.

where K is known as the Hydraulic Conductivity [LT-1].

Notice here that the hydraulic conductivity is given by:

$$K = \frac{k}{\mu} \rho g$$

and is property both of the porous medium, since it is a function of *k*, and of the fluid contained in it, since it depends also on μ

2.6 Flow in anisotropic porous media (constant density)

What is the effect of anisotropy on the flow of water through the rock and how can it be described by Darcy's law?

To answer this question qualitatively, we might consider the case of a rock containing a large number of open fractures all aligned in approximately the same direction, which allow significant quantities of water to pass through them easily. Suppose also that the unfractured rock matrix between them has a comparatively low hydraulic conductivity.

It follows that if we apply a force to the water in the rock, there will be a greater tendency for the water to move parallel to the fracturing (rather than across it) even though the applied force may not be in that direction. This is the essential result which applies to flow in all anisotropic media, i.e. that the water does not necessarily move in the direction of the applied force (the hydraulic gradient).

For simplicity, we concentrate here on the case in which flow is anisotropic but not density dependent. It can be shown that, for any natural system, there are three special directions, known as the principal directions of hydraulic conductivity, which are at right angles to each other. It can also be shown that if the coordinate axes are aligned with these principal directions, then Darcy's Law is:

$$q = -\left(K_x \frac{\partial h}{\partial x} i + K_y \frac{\partial h}{\partial y} j + K_z \frac{\partial h}{\partial z} k \right)$$

2.7 The mass continuity equation

The principal of the conservation of mass is fundamental to all hydrology and is expressed mathematically by the continuity equation.

The fundamental principle is that, for a fixed volume over a given period of time:

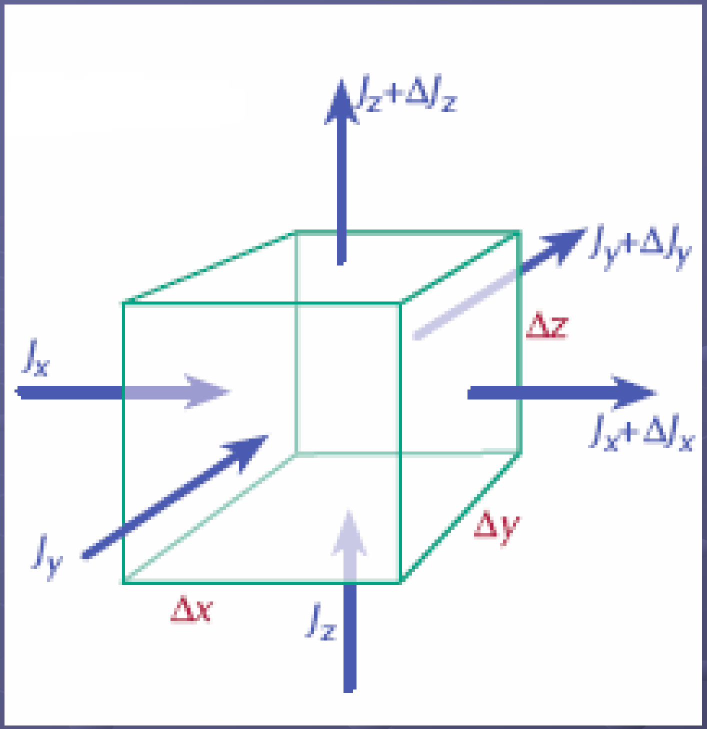
increase in mass stored = mass inflow – mass outflow

the following figure shows the mass fluxes across the walls of the control volume. For example, (*Jx)* is the mass per unit area per unit time entering the control volume parallel to the *x*-axis and (*Jx* +Δ*Jx)* is the mass per unit area per unit time leaving the control volume in the same direction.

We define (*M*) to be the mass of the substance of interest per unit volume, and (Δ*M)* the increase in mass per unit volume in time Δ*t*.

The total increase in mass in the volume in time Δ*t* is, therefore, $\Delta M \Delta x \Delta y \Delta z$.

We also define the net increase in mass (per unit volume per unit time) due to internal sources and sinks by (*N)* . In groundwater terms, a sink might be an abstraction from a well whose intake lies within the control volume.



$$\Delta M \Delta x \Delta y \Delta z = -\Delta Jx \Delta y \Delta z \Delta t - \Delta Jy \Delta z \Delta x \Delta t - \Delta Jz \Delta x \Delta y \Delta t + N \Delta x \Delta y \Delta z \Delta t$$

Dividing through by $\Delta x \Delta y \Delta z \Delta t$ *gives*:

$$\frac{\Delta M}{\Delta t} = -\left(\frac{\Delta Jx}{\Delta x} + \frac{\Delta Jy}{\Delta y} + \frac{\Delta Jz}{\Delta z} \right) + N$$

Each of the terms in previous *Equation* are finite difference approximations to derivatives, and letting Δx, Δy, Δz and Δt tend to zero gives the continuity equation:

$$\frac{\partial M}{\partial t} = -\left(\frac{\partial Jx}{\partial x} + \frac{\partial Jy}{\partial y} + \frac{\partial Jz}{\partial z} \right) + N$$

The bracketed expression in the last *Equation* is called the divergence of the mass flux vector J, and is sometimes written more compactly as ∇J; thus it can also be written as:

$$\frac{\partial M}{\partial t} = -\nabla . J + N$$

2.8 The storage of fluid within the Aquifer

- Aquifers act both as water conduits and as reservoirs. Under steady state flow conditions the quantity of water stored in the aquifer remains constant in time.

- Water flowing from a volume of aquifer is replaced by an equal quantity of incoming water. However, recharge to an aquifer and abstractions from it are generally time varying. In such cases the amount of water stored in the aquifer changes over time.

- When water is pumped from a phreatic aquifer, water is released from storage by the lowering of the water table, and the volume of water released is equal to the volume of extra void spaces produced by this lowering.

- **When a confined aquifer is pumped, water is released due to the effects of reducing the pressure within the aquifer.**
- **This reduction in pressure may result in the compaction of the aquifer, reducing the void space and squeezing out the groundwater like water from a sponge.**
- **If the pressures in the aquifer are high enough to compress the groundwater itself then reducing the pressure will result in the groundwater increasing in volume, causing an increased yield.**
- **If the pizeometric surface is drawndown below the top of the aquifer then the aquifer will behave locally like a phreatic one.**

We define the specific storativity S_s [L^{-1}] by

$$S_s = \rho g [\alpha + n\beta]$$

α **is the coefficient of compressibility of the aquifer**

β **is the coefficient of compressibility of the fluid**

3. Compressibility and Effective Stress

- The analysis of ***transient groundwater flow*** requires the introduction of the concept of compressibility.

- ***Compressibility*** is a material property that describes the change in volume, or strain, induced in a material under an applied stress.

- In the classical approach to the strength of elastic materials, the ***modulus of elasticity*** is a more familiar material property. It is defined as the ratio of the ***change in stress $d\sigma$ to the resulting change in the strain $d\varepsilon$.***

- **Compressibility is simply the *inverse of the modulus of elasticity*. It is defined as *strain/stress, dε/dσ,* rather than stress/strain, *dσ/dε*.**

- **For the flow of water through porous media, it is necessary to define two compressibility terms, one for the water and one for the porous media.**

3.1 Compressibility of water (fluid)

- **Compressibility of water, β can be defined as:**

$$\beta = -\frac{dV_w / V_w}{dP}$$

- **The negative sign is necessary if we wish β to be a positive number.**

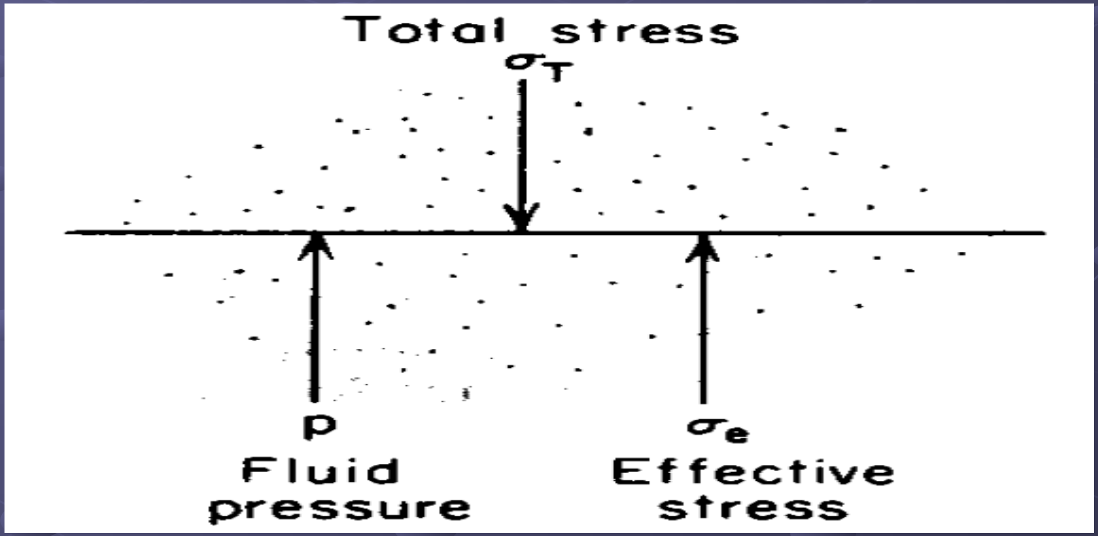
- An increase in pressure *d*p leads to a decrease in the volume Vw of a given mass of water.
- $\frac{dV_m}{V_w}$: volumetric strain induced by *d*P
- Note that V=Mass/Density

$$\beta = \frac{d\rho / \rho}{dP}$$

- β = zero for an incompressible fluid.

3.2 Effective Stress

- **Assume that a stress is applied to a unit of saturated sand. There are three mechanisms behind a reduction in the volume:**
  - **Compression of the water in the porous (*β*).**
  - **Compression of the water in the individual sand grains**
  - **Rearrangement of the sand grains into a more closely packed configuration.**



σT : total stress due to weight of overlying rock and water.

- **The is an upward stress caused by fluid pressure and the actual stress that is born by aquifer skeleton, the latter is called effective stress.**

- Total stress, effective stress, and fluid pressure are related by the simple equation:

$$\sigma_T = \sigma_e + P$$

or, in terms of the changes,

$$d\sigma_T = d\sigma_e + dP$$

- The weight of rock and water overlying each point in the system often remains essentially constant through time.

$$d\sigma_T = 0 \Rightarrow d\sigma_e = -dP$$

- If the fluid pressure increases, σ_e decreases by equal amount.
- If the fluid pressure decreases, σ_e increases by equal amount
- When pumping an aquifer occurs:
  - the fluid pressure decreases and σ_e increases.
  - Aquifer skeleton may compact
  - From definition of β, the volume of water in aquifer will expand.

3.3 Aquifer Compressibility

- The compressibility of a porous media is defined as:

$$\alpha = -\frac{dV_T / V_T}{d\sigma_e}$$

- So, if σe increase the total volume of a soil mass decreases.

$$\alpha = -\frac{db / b}{d\sigma_e}$$

- The aquifer compressibility can be defined as

- db: change in aquifer thickness.

  The negative sign indicates that the aquifer gets smaller with an increase in effective stress.

- Since

$$d\sigma_e = -dP \Rightarrow \alpha = \frac{db/b}{dP}$$

When a well in an aquifer is being pumped then:

- **Fluid pressure decreases, and so σe increases by equal amount. Aquifer will be compact by *db.***
- **The fluid pressure increases, σe decreases by equal amount. Aquifer expand.**

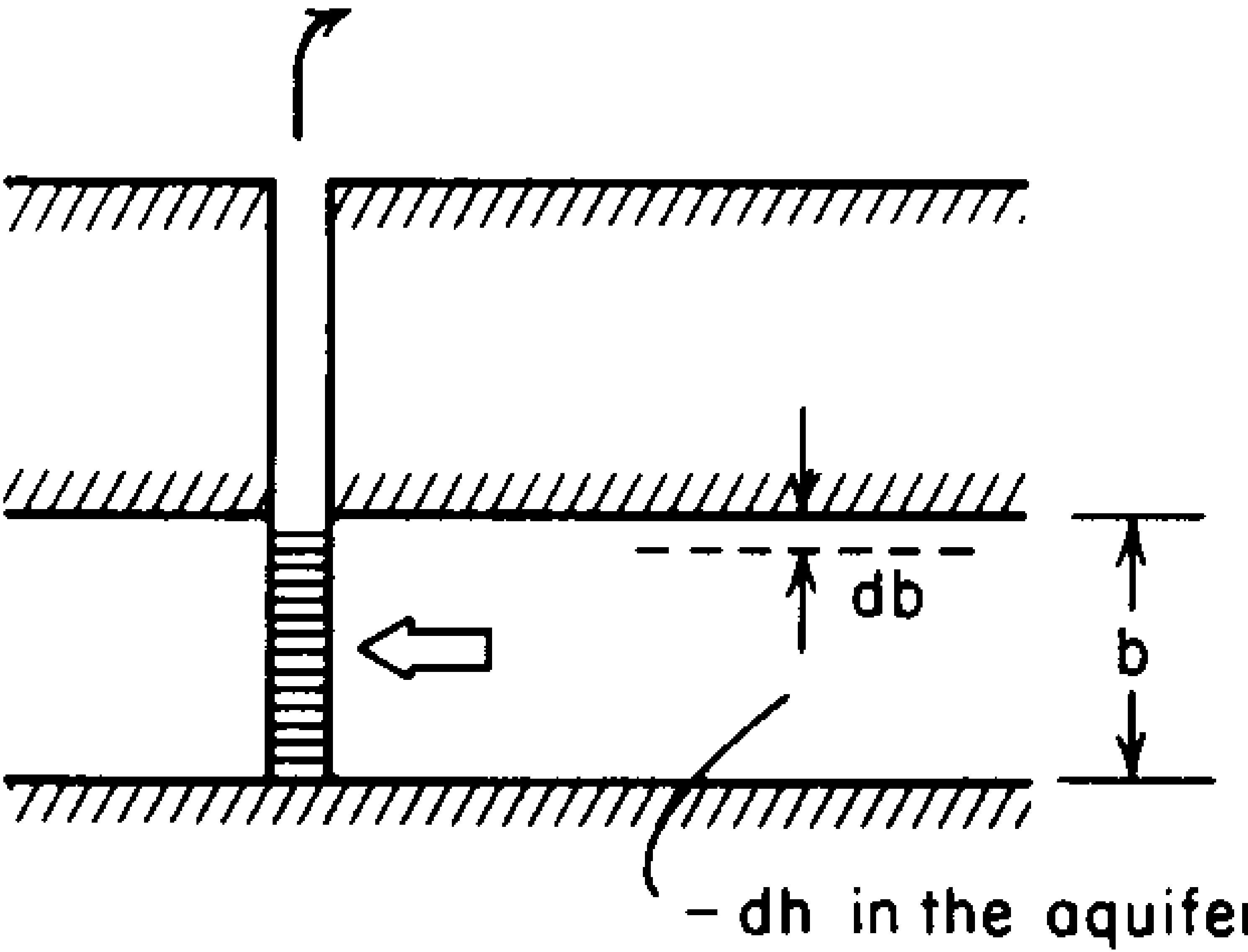

db
b
– dh in the aquifer

Table 4 shows typical values of α which are given by Freeze and Cherry for a variety of rocks

Formation	α $m^2 N^{-1}$
✓ Clay	$10^{-8} – 10^{-6}$
✓ sand	$10^{-9} – 10^{-7}$
✓ gravel	$10^{-10} – 10^{-8}$
✓ jointed rock	$10^{-10} – 10^{-8}$
✓ sound rock	$10^{-11} – 10^{-9}$

THANK YOU